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Bifurcation of stationary manifolds formed in the neighborhood of the equilibrium in a dynamic system of cutting

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ABSTRACT

The problems related to nonlinear dynamics of material processing by cutting are reviewed in this study. A mathematical model of a dynamic system that considers the dynamic link, formed by the cutting process, is proposed. The following key features of the dynamic links are examined: the dependence of the cutting forces on the area of the shear layer, lag of forces with respect to the elastic deformation displacement of the tool relative to the workpiece, the restrictions imposed on the movement of the tool toward the rear end of the instrument with the treated part of the workpiece, the dependence of the forces on the cutting speed, and the change of force components at varying angles of the tool with respect to the direction of movement of the tool relative to the workpiece.

The dynamic subsystem of the tool is presented by a linear dynamic system in the plane normal to the cutting surface. The focus of this study is on the analysis of attracting sets formed near the equilibrium point (orbitally asymptotically stable limit cycles, two-dimensional invariant tori, and chaotic attractors). It is shown that by considering the bending deformation of the tool, there is a possibility of branching of equilibrium points during changes of control parameters. Data on the bifurcations of the parametric space and the space of control parameters are shown. The general laws of buckling equilibrium of the system are reviewed.

1. Introduction

The dynamic system under investigation comprises cutting tool and workpiece subsystems, which are combined with a dynamic link formed by machining processes [1–11]. This relationship, which represents a model of cutting forces in the coordinates of the state of interacting subsystems, is nonlinear and identifies the main features of the system. There is a possible loss of stability of the equilibrium point, which is observed in the moving coordinate system, whose movement, when turning, is determined by the trajectories of the actuators of the machine (support) and rotation of the spindle [5,8,12]. If the equilibrium is unstable, a variety of attracting manifolds are formed in its neighborhood (e.g., limit cycles, invariant tori, and chaotic attractors) [6–8,10–17,19,20].

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Cutting forces depend on the area that is formed by the intersection of the tool and the workpiece, that is, from the trajectories of the actuators of the machine, tool geometry, and elastic deformation displacement of the tool and the workpiece. In a previous study, geometric parameters of the cutting tool in the observed moving coordinate system are considered constant and independent of the trajectories of elastic deformation displacement. Laws of the transformation of the area to the forces are determined by many physical processes, which change over time. Therefore, there are phase displacements between forces and deformations [1–6,9,13]. The presence of phase shifts is modeled by aperiodic links, link of pure delay, or the introduction of hysteretic relationship between displacements and forces [5,6,13–15].

The turnaround delay due to variations in the area of the shear layer at the previous turnaround is examined. In many cases, particularly in milling, the parameters in the constraint equations are considered as periodic functions of time [21–23]. The Mathieu–Hill equation and Floquet theory are used to study systems with periodically varying parameters [24,25]. As a rule, simple scalar (not spatial) models, where the existence of chaotic attractors is confirmed, are used [26,26–29].

Moreover, the diversity of models described above is observed experimentally and in cases where the periodic changes in the parameters are explicitly unavailable. In addition, an increase in the rate determined by the speed of the machine actuators and elastic deformation leads to a reduction of forces [16–20]. In this case, a modified equation of Van der Pol and Rayleigh is used [18]. These models reveal the dependence of forces on the current value of the tangential speed of movement of the tool relative to the detail, and they do not consider the variation of the area of the shear layer [21]. In Refs. [19,20] the dependence of forces of the variation of the shear layer and the dependence of friction force on the speed in two orthogonal directions are studied. The force generated in the cutting area and the frictional forces are independent. This model also leads to the formation of various attracting sets in the neighborhood of equilibrium of the system.

All the studies cited above analyzed only one of the following three mechanisms of loss of stability: the effect of the delay, the relationship between forces and speed, or periodic changes in the parameters of the equation. In many cases, the system is examined as the sole oscillating link. A real system is examined as a nonlinear effect-specific circuit with several interrelated mechanisms of loss of stability. This work is novel because of the consideration of the spatial model of the elastic system, with a diversity of simultaneously existing sources of self-excitation. For example, gyroscopic, circulation, and accelerating forces in a spatial model varying with respect to the equilibrium point occur naturally as a reaction from the treatment process. These forces characterize the general properties and mechanisms of loss of stability. However, they have not been examined so far.

Moreover, the influence of angular deformation shifts of the cutting tool on forces was not examined. For example, the increase of the amplitude of periodic movement of the tool in the direction of the forming surface and the value of angle clearance of the tool will not only decrease but also have a negative value, causing a rapid disproportionate increase of forces acting on the rear end of the tool. These forces, which are dependent on the velocity of the elastic vibrational displacements, are directed against the movement, hence causing a nonlinear dissipation of the process. If the elastic deformation of the tool in the direction of the cutting speed leads to not only a shift in the instrument but also its rotation, then the increase in the forces takes place without altering the area of the shear layer. This is due to the dependence of the forces on the rake angle of the cutting tool.

Thus, the positive feedback between the strains and forces that are dependent on the elastic deformation causes not only additional conditions for self-excitation but also branching of equilibrium points of the system. The simultaneous consideration of several interrelated conditions of self-excitation of the system, the consideration of the effect on the dynamics of the variations of cutting angles, depending on the strain displacements, and the disclosure of some of the general laws of force generation are characterized by the further development of the main features of the nonlinear dynamics of the cutting process. To the best of the authors' knowledge, this is a new result.

2. Basic mathematical model

The main properties of the system can be expanded if a simplified model of the process of free cutting is used. Without revealing the force structure, the dynamic equation can be written as [9,32] (Fig. 1)

$$m\frac{\mathrm{d}^2X}{\mathrm{d}t^2}+h\frac{\mathrm{d}X}{\mathrm{d}t}+cX=F(t), \tag{1}$$

where $m=\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}$, $h=\begin{bmatrix} h_{1,1} & h_{2,1} \\ h_{1,2} & h_{2,2} \end{bmatrix}$, and $c=\begin{bmatrix} c_{1,1} & c_{2,1} \\ c_{1,2} & c_{2,2} \end{bmatrix}$ are the symmetric and positively definite matrix of inertia, and dissipative and elastic coefficients of the subsystem tool; $X=\{X_1,X_2\}^T$ is the elastic deformation displacement vector in a plane normal to the cutting surface (in the direction A–A1 in Fig. 1); and $F=\{F_1(t),F_2(t)\}^T$ is the vector of the cutting forces. Deformation changes are considered in the coordinate system moving in the direction of the workpiece at a constant speed of the support V_C. Angular velocity of rotation of the workpiece (Ω) is constant; therefore, for one rotation, T=const. Furthermore, the set speed of cutting (V_C) is also constant. Therefore, in the steady state, the flow value ($V_CT = S_P$) is constant. The cutting force is calculated as the sum of the forces acting on the front and rear ends of the instrument, that is, $F=F^{(1)}+F^{(2)}$ (Fig. 1). Parameters m, h, and c can be determined by the methods described, for example, in Refs. [9,33].