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Analytical modeling of large amplitude free vibration of non-uniform beams carrying a both transversely and axially eccentric tip mass

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ABSTRACT

The objective of this paper is to study large amplitude flexural–extensional free vibration of non-uniform cantilever beams carrying a both transversely and axially eccentric tip mass. The effects of variable axial force is also taken into account. Hamilton's principle is utilized to obtain the partial differential equations governing the nonlinear vibration of the system as well as the corresponding boundary conditions. A numerical finite difference scheme is proposed to find the natural frequencies and mode shapes of the system which is validated specifically for a beam with linearly varying cross section. Using a single mode approximation in conjunction with the Lagrange method, the governing equations are reduced to a set of two nonlinear ordinary differential equations in terms of end displacement components of the beam which are coupled due to the presence of the transverse eccentricity. These temporal coupled equations are then solved analytically using the multiple time scales perturbation technique. The obtained analytical results are compared with the numerical ones and excellent agreement is observed. The qualitative and quantitative knowledge resulting from this research is expected to enable the study of the effects of eccentric tip mass and non-uniformity on the large amplitude flexural–extensional vibration of beams for improved dynamic performance.

1. Introduction

Many components of important engineering structures can be idealized as beams carrying a concentrated mass at its free end. Some examples are flexible robot arms, water towers, wind tunnel stings, aircraft wings carrying external tanks or missiles, atomic force microscopes, most antenna structures and so on. The presence of the tip mass plays an important role in the dynamic characteristics of the beam and exert inertial force which is a function of motion itself. This problem has been received many attention in the prior art (for example see [1–3]). Most of the researchers assumed the beam to be uniform. However, it is well-known that in many cases, non-uniform beams may achieve a better distribution of strength and weight, and sometimes can satisfy special architectural and functional requirements [4]. The non-uniformity which may arise from variable cross section or inhomogeneous material properties leads to a fourth order partial differential

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equation (PDE) with variable coefficients and in general, cannot be analytically solved. However for some specific non-uniform beams, exact solutions for the eigen-value problem are reported in terms of Bessel functions [5–7], orthogonal polynomials [8], hypergeometric series [9] and power series [10].

In reality, the centroid of the tip mass will not coincide with its attachment point to the beam. For a uniform beam with axially eccentric tip mass, Bhat and Wagner [11] derived an approximate solution for the frequency equation using perturbation technique. To [12] derived an exact expression for natural frequencies and mode shapes of a uniform beam with base excitation and axially eccentric tip mass. Auciello [13] suggested an exact analysis of free vibration of a linearly tapered cantilever beam with tip mass of rotary inertia and axial eccentricity. In some applications, the eccentricity of the tip mass can exist in both transverse and axial directions. As an example, in the case of wind turbine towers, the nacelle can be modeled as a tip mass which is obviously eccentric in both transverse and axial directions. This condition may also be observed in the modeling of aircraft wings carrying external tanks or missiles. In such a condition, the longitudinal and transverse motions of the beam are coupled which raises the complexity of the problem.

For small amplitude oscillations, the response of a beam can be adequately described by linear equations. However, as the amplitude of oscillations increases, geometric nonlinear effects come into play. The geometric nonlinearity may be caused by nonlinear stretching or large curvature. Nonlinear stretching of the midplane of a beam leads to a nonlinear relationship between the strain and the displacement. If the large amplitude vibrations are accompanied by large changes in the curvature, the nonlinear dependence of the curvature to the displacement field shall be considered [14]. Zavodney and Nayfeh [15] investigated the nonlinear response of a slender cantilever beam carrying a lumped mass and rotary inertia to a principal parametric base excitation. The multiple time scales (MTS) method used to determine an approximate solution of the temporal equation. Dwivedy and Kar [16] investigated the nonlinear response of a cantilever beam carrying a lumped mass at an arbitrary position with 1:3:9 internal resonances. Hamdan and Shabaneh [17] studied the large amplitude free vibration of a slender, inextensible cantilever beam with a rotationally flexible root, carrying a lumped mass at an intermediate position along its span. While the influence of axial inertia and nonlinear curvature were considered in their study, shear deformations and rotary inertia effects were neglected. These effects which are included in Timoshenko beam theory, become more significant for non-slender or short beams and for higher modes of vibration. Based on Timoshenko beam theory, Dias [18] performed a general exact harmonic analysis of in-plane beam structures using classical dynamic stiffness matrix. Dias and Alves [19] addresses the same problem in the presence of end releases and rigid offsets. Rao et al. [20] applied the coupled displacement field method for the large amplitude free vibrations of uniform Timoshenko beams with central concentrated mass. Esmailzadeh and Jalili [21] investigate the parametric response of a cantilever beam with tip mass subjected to harmonic support motion.

In the present study, large amplitude free vibration of a non-uniform vertical cantilever beam carrying an axially and transversely eccentric tip mass with rotary inertia is considered. As the beam is assumed to be under the effect of gravity, the spatial variation of axial force is taken into account. Considering variable properties for the beam, geometric nonlinearities arose from large deflections and Euler–Bernoulli's assumptions, the equations of motion and the corresponding boundary conditions are derived using Hamilton's principle. In order to find the natural frequencies and mode shapes of the beam with generally variable physical and geometrical properties, a numerical finite difference scheme is presented. The proposed method can also model the effect of eccentric tip mass while including the variable axial force. Using a single mode approximation, the ordinary differential equations (ODEs) governing the large amplitude oscillations of the beam are obtained. The method of MTS is then utilized to find closed-form expressions for dynamic response of the beam. Finally the results are compared with numerical simulations and good agreement is observed.

2. Problem formulation

The schematic view of a non-uniform beam with an eccentric tip mass is shown in Fig. 1. This system can represent the tower and nacelle of a wind turbine. In this figure, $\hat{a}$ and $\hat{b}$ are the transversal and axial eccentric values respectively, CG is the center of gravity of the tip mass $\hat{M}$, and l is the length of the beam. Furthermore, $\hat{J}$ is the polar mass moment of inertia of the tip mass around CG. In the modeling process presented in this section, the axial stretching of the beam due to the gravitational and inertial axial forces will be taken into account.

In order to find the governing equations of motion of this system, Hamilton's principle will be used. According to this principle, every system behaves in such a way that the following equation is satisfied:

$$\delta \int_{\hat{t}_1}^{\hat{t}_2} \left(\hat{K} - \hat{U} + \hat{W}_{Ext}\right) \mathrm{d}\hat{t} = 0 \tag{1}$$

where δ is the variation operator, $\hat{t}_1$ and $\hat{t}_2$ are arbitrary times, $\hat{K}$ and $\hat{U}$ are the total kinetic and potential energies of the system respectively and $\hat{W}_{Ext}$ is the work done by external forces.

The Euler–Bernoulli's assumptions is considered here. In the Euler–Bernoulli or thin beam theory, the rotation of cross sections of the beam is neglected compared to its translation. In addition, the angular distortion due to shear is considered negligible compared to the bending deformation. These assumptions are applicable for long and slender beams with length much greater than the thickness [22]. Assuming the beam undergoes large deflections, the nonlinear strain–displacement