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# Point group identification algorithm in dynamic response analysis of nonlinear stochastic systems

Tao Li [a,b], Jian-bing Chen [b], Jie Li [b]

[a] School of Environment and Architecture, University of Shanghai for Science and Technology, 516 Jungong Road, 200093 Shanghai, China
[b] School of Civil Engineering, Tongji University, Siping Road 1239, 200092 Shanghai, China

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ABSTRACT

The point group identification (PGI) algorithm is proposed to determine the representative point sets in response analysis of nonlinear stochastic dynamic systems. The PGI algorithm is employed to identify point groups and their feature points in an initial point set by combining subspace clustering analysis and the graph theory. Further, the representative point set of the random-variate space is determined according to the minimum generalized F-discrepancy. The dynamic responses obtained by incorporating the algorithm PGI into the probability density evolution method (PDEM) are compared with those by the Monte Carlo simulation method. The investigations indicate that the proposed method can reduce the number of the representative points, lower the generalized *F*-discrepancy of the representative point set, and also ensure the accuracy of stochastic structural dynamic analysis.



## 1. Introduction

Stochastic dynamics has gained increasing attention from researchers in many scientific fields. The randomness of stochastic dynamical systems comes from external excitations as well as structural properties [1]. Structural systems are generally at nonlinear state when they are subjected to severe external excitations [2]. Due to the coupling of randomness and nonlinearity, the dynamic response of a stochastic system is a random variable or a random process. The more effective method is to capture the probabilistic information of structural dynamic responses [3]. The methods of moment and solving the FPK equation were developed extensively in the field of random vibration. The dynamic responses of a linear stochastic system can be attained by the moment method [4,5]. But the responses of nonlinear stochastic systems cannot be investigated conveniently by this approach because of the non-closed problem of the moment equations [5,6]. While the Monte Carlo simulation is always concerned because of its versatility, it is seldom applied in practical large engineering structures due to its prohibitive computational burdens. To solve the problem of compound random vibration involving the randomness from structural properties and external excitations, the Monte Carlo simulation and generalized chaos polynomial method are employed generally [7–9]. Combining the random collocation point method, the investigation of a stochastic dynamic system can be achieved by using the generalized chaos polynomial method [10,11]. Nevertheless, the number of the orthogonal points of the polynomial grows exponentially with the increase of the number of random variables. In the situation, the orthogonal chaos polynomial method is not of acceptable efficiency, namely, it also needs to face the performance impact of the dimensionality curse.

*E-mail addresses:* nmbtlitao@163.com (T. Li), Chenjb@mail.tongji.edu.cn (J.-b. Chen), Lijie@mail.tongji.edu.cn (J. Li).

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In the past nearly 10 years, a new approach named the probability density evolution method (PDEM) proposed by Li and Chen has been developed systematically [12–16]. It could capture the instantaneous probability density function (PDF) of responses of general nonlinear MDOF structures with the randomness involved in structural properties and external excitations. In the PDEM, a generalized density evolution equation (GDEE) is derived and can be solved by the numerical method. The first step to solve the GDEE is the selection of representative point set based on the partition of probability-assigned space [17]. In the PDEM, a representative point set could be picked out by the Grid-type point method or tangent sphere method, if the number of random variables is small [18,19]. Otherwise a representative point set could be selected by using the number theoretical method, quasi-symmetric point method and the crude Monte Carlo [18,20,21]. Because of the sparsity and symmetry of points selected by the quasi-symmetric point method, the projections of these points on each dimension of random-variate space overlap each other seriously. Therefore, the information of marginal probability density of random variables cannot be reflected adequately [22]. In the number theoretical method, the representative points were selected further by a hyper-sphere. The points inside the hyper-sphere are picked out and the rest are rejected. Although the number of representative points is reduced effectively, the calculation error increases with the dimensionality of random-variate space increasing. Although the number of representative points selected by the number theoretic method is far less than that needed by the stochastic random number method, as presented in [18], it must be large enough to ensure the computational accuracy of stochastic dynamic analysis.

In the paper, the point group identification (PGI) algorithm is proposed. This algorithm combines the ideas of subspace clustering techniques, e.g. the technique of clustering in quest (CLIQUE) [23], and the graph theory. Clustering techniques have been studied extensively in statistics, pattern recognition, bioengineering and data analysis [24–26]. Based on the subspace clustering techniques and graph theory, this algorithm can identify point groups in the random-variate space and determine the feature points to constitute the representative point set. Each feature point carries the information of all the points in an identified point group represented by it. The probability density solutions of the dynamic responses of stochastic structures in which the randomness comes both from structural parameters and external excitation can be attained by incorporating the PGI algorithm into the PDEM. The efficiency and accuracy of the proposed approach is verified through the numerical examples.

We first provide a brief review of the PDEM in Section 2 which is the foundation of the algorithm PGI. In Section 3 the GF-discrepancy is introduced, which is used as a metric of the homogeneity of a point set in the paper. The efficiency of the algorithm PGI can be verified by the GF-discrepancy. The algorithm PGI is proposed in Section 4. The numerical examples are adopted to demonstrate the efficiency and accuracy of the algorithm PGI in Section 5. In Section 6 we conclude by discussing the advantages of the algorithm PGI and an outlook of the further work.

## 2. Probability density evolution method

Considering a generic stochastic dynamical system, the equation of motion is as follows [27]:

$$\dot{\mathbf{Y}} = G(\boldsymbol{\Theta}, \mathbf{Y}, t) \tag{1}$$

where $\mathbf{Y} = (Y_1, Y_2, ..., Y_n)$ is a state vector including displacements and velocities of the dynamic system; $\mathbf{Y}_0$ is the initial vector; $n$ is the dimension of the dynamical system; $\boldsymbol{\Theta} = (\Theta_1, \Theta_2, ..., \Theta_s)$ is an $s$-dimensional random vector involved with known joint probability density function $p_{\boldsymbol{\Theta}}(\boldsymbol{\theta})$. $\Theta_j (j = 1, 2, ..., s)$ are random variables representing the randomness either from excitations or system parameters. In some cases, random excitations or random system parameters are modeled as random processes, which can be further expressed by a combination of a series of random variables, say through some decomposition methods of random process, e.g. the Karhunen–Loeve decomposition method [34] and orthogonal decomposition method [27]. Namely, the random process should firstly be represented by some random variables. Then these random variables together with other random variables become the components of the random vector $\boldsymbol{\Theta}$. $\boldsymbol{\Theta}$ is still a random vector consisting of random variables, but the dimensionality of the random vector $\boldsymbol{\Theta}$ increases.

If a random-variate space $\Omega_{\boldsymbol{\Theta}}$ is divided into some non-overlapping partitioning subspace $\Omega_q$, $q = 1, 2, ..., n_{pt}$, and

$$P_q = \int_{\Omega_q} p_{\boldsymbol{\Theta}}(\boldsymbol{\theta}) d\boldsymbol{\theta} \tag{2}$$

is the assigned probability over $\Omega_q$ [21]. The advanced probability density evolution equation is proposed [16]:

$$\frac{\partial p_q(y,t)}{\partial t} + \int_{\Omega_q} \left( \dot{Y}(\boldsymbol{\theta}, t) \frac{\partial p_{Y\boldsymbol{\Theta}}(y, \boldsymbol{\theta}, t)}{\partial y} \right) d\boldsymbol{\theta} = 0 \tag{3}$$

where $p_{Y\boldsymbol{\Theta}}(y, \boldsymbol{\theta}, t)$ is the joint probability density function of $(\mathbf{Y}(t), \boldsymbol{\Theta})$. It is denoted that:

$$p_q(y,t) = \int_{\Omega_q} p_{Y\boldsymbol{\Theta}}(y, \boldsymbol{\theta}, t) d\boldsymbol{\theta}, \quad q = 1, 2, ..., n_{pt} \tag{4}$$

where functions $p_q(y,t)$, $q = 1, 2, ..., n_{pt}$, are not probability density functions because $\int_{-\infty}^{+\infty} p_q(y,t) dy = P_q \neq 1$. However, after defining a normalized function $\tilde{p}_q(y,t) = p_q(y,t)/P_q$, the consistency condition is satisfied: $\int_{-\infty}^{+\infty} \tilde{p}_q(y,t) dy = 1$. The function