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Multi-parametric analysis of the lowest natural frequencies of strongly inhomogeneous elastic rods

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ABSTRACT

The results of a multi-parametric analysis of the near-rigid body motions of a three-component strongly inhomogeneous elastic rod are presented. It is demonstrated that the values of the associated lowest natural frequencies tend to zero at large/small ratios of material and geometric parameters. The low-frequency behaviour is classified into global and local regimes and the general conditions supporting global low-frequency regimes are derived. As an example, a rod with piecewise uniform properties is considered. A perturbation procedure oriented to a more general setup is developed.

1. Introduction

Vibrations of inhomogeneous elastic structures find numerous applications in modern engineering, see e.g. [1,2] among other contributions. In particular, we mention vibration tailoring, i.e. design of elastic structures with specified values of natural frequencies [3,4].

In many cases structural component parts may have contrast material and geometric characteristics, including stiffness, volume density and size. As a typical example, we mention sandwich plates which are widely used in civil and mechanical engineering, and are therefore intensively studied, see e.g. [5–7], and references therein. The number of potential applications may increase significantly due to a rapidly growing area of metamaterials, see e.g. [8,9].

This paper is concerned with investigation of low frequency vibrations of elastic rods composed of parts with contrast properties. It is observed that under certain restrictions on the material parameters and lengths the lowest natural frequencies become small, tending to zero at the limit of large/small contrasts.

The analysis is carried out for free vibrations of a non-uniform three-component rod. The basic problem parameters involve the ratios of Young's moduli, densities and lengths of the components. A multi-parametric treatment reveals two types of low-frequency motions. One of these is the so-called global low-frequency regime corresponding to quasi-static behaviour of all of the rod's components. Another type may be referred to as local low-frequency regime describing quasi-static behaviour of “stronger”, in particular, stiffer components and not preventing oscillating profiles in “weaker” parts, which is similar to recent results for homogenization of contrast periodic media, see [10,11].

In the paper the attention is first drawn to the model problem of low-frequency vibrations of a symmetric piecewise uniform rod with free ends, allowing a straightforward analytical treatment. The conditions on the parameters supporting

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the global low-frequency regime are derived. The associated displacement profile is approximated at leading order by linear functions. This agrees with an intuitive expectation that the "stronger" parts perform almost rigid body motion.

The consideration is then extended to the local low-frequency behaviour, along with other types of boundary conditions supporting global low-frequency regime. The cases which do not allow global low frequency modes are mentioned. Finally, the analysis is generalized to a non-uniform three-component rod with variable material parameters depending on the longitudinal coordinate. Since a straightforward analytical approach is no longer possible, a perturbation scheme is developed leading to estimates for low natural frequencies and the displacement profile.

The developed methodology is not restricted to the three-component rod considered in the paper. It has a potential to be extended for a more sophisticated setup. In particular the obtained results could be applied to the analysis of the lowest cut-off frequencies [12,13] of sandwich plates and shells and also have some more general implications to 2D and 3D dynamic problems for elastic structures with contrast material parameters. In the latter case the perturbation scheme presented in this paper may need to be amended by including numerical routines.

2. Basic equations

Consider time-harmonic vibrations of an elastic rod composed of three inhomogeneous parts. The axis Ox is chosen such that the origin O is located in the middle of the inner part. The rod is finite, with the outer parts having free or fixed ends, and continuity assumed between the components. The inner part of the rod occupies the region $|x| \le h_1$, with the outer parts specified by $-h_1-h_3 \le x \le -h_1$ and $h_1 \le x \le h_1+h_2$ (Fig. 1).

The governing equations are written in the form

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(E_i\frac{\mathrm{d}u}{\mathrm{d}x}\right)+\rho_i\omega^2 u=0, \quad i=1,2,3, \tag{1}$$

where u is the displacement, E_i are Young's moduli, ρ_i are the material densities, $c_i=\sqrt{E_i/\rho_i}$ are the longitudinal wave speeds and ω is the vibration frequency. Here the indices 1, 2 and 3 correspond to the inner, right outer and left outer parts, respectively.

The continuity of displacements and stresses at the interfaces $x=\pm h$ is assumed

$$\begin{aligned} u(-h_1+0)&=u(-h_1-0),\\ u(h_1-0)&=u(h_1+0),\\ E_1u'(-h_1+0)&=E_3u'(-h_1-0),\\ E_1u'(h_1-0)&=E_2u'(h_1+0). \end{aligned} \tag{2}$$

We consider three types of boundary conditions on the outer ends, namely free ends, with zero stresses prescribed on both ends of the rod

$$\begin{aligned} u'(-h_1-h_3)&=0,\\ u'(h_1+h_2)&=0, \end{aligned} \tag{3}$$

fixed ends, with zero displacements imposed on the ends

$$\begin{aligned} u(-h_1-h_3)&=0,\\ u(h_1+h_2)&=0, \end{aligned} \tag{4}$$

and mixed boundary conditions, being a combination of the previous two cases

$$\begin{aligned} u(-h_1-h_3)&=0,\\ u'(h_1+h_2)&=0. \end{aligned} \tag{5}$$

The variable material parameters may be represented as

$$E_i=E_i^*\tilde{E}_i(x), \quad \rho_i=\rho_i^*\tilde{\rho}_i(x), \quad c_i=c_i^*\tilde{c}_i(x), \; i,j=1,2,3, \tag{6}$$

where E_i^*, ρ_i^* and c_i^* are average values of the associated quantities. We also introduce dimensionless longitudinal variable

$$\chi=\frac{x}{h_1}, \tag{7}$$

and scaled frequencies

$$\lambda_i=\frac{\omega h_i}{c_i^*}, \quad i=1,2,3, \tag{8}$$

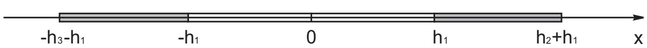

Fig. 1. Geometry of the 3-component inhomogeneous rod.