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Critical velocity of a uniformly moving load on a beam supported by a finite depth foundation

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ABSTRACT

In this paper, a new formula for the critical velocity of a uniformly moving load is derived. It is assumed that the load is traversing a beam supported by a foundation of a finite depth. Simplified plane models of the foundation are presented for the analysis of finite and infinite beams, respectively. Regarding the model for finite beams, only the vertical dynamic equilibrium is considered. Then the critical velocity obeys the classical formula with an augmented mass that adds 50% of the foundation mass to the beam mass. In the model for infinite beams, the effect of shear is added in a simplified form and then the critical velocity is dependent on the mass ratio defined as the square root of the fraction of the foundation mass to the beam mass. For a low mass ratio, the critical velocity approaches the classical formula and for a higher mass ratio, it approaches the velocity of propagation of shear waves in the foundation. The formula can also account for the effect of the normal force acting on the beam. Deflection shapes of the beam are obtained semi-analytically and the influence of different types of damping is discussed.

1. Introduction

Structures subjected to moving loads have several applications in various areas of transport engineering. The critical velocity of the moving load stands for an important indication of the viability and safety of the referred structures. For instance, in the area of high-speed railway transportation, a theoretical concept that is based on the assumption that the track structure acts as a continuously supported beam resting on a uniform layer of closely spaced, mutually independent, linear elastic vertical springs, providing resistance in direct proportion to the deflection of the beam and representing the underlying remainder of the track structure, can be introduced. The stiffness of such spring layer along the length of the track is named as the track modulus and defines the Winkler model, often referred to as a “one-parameter model”. Then the critical velocity of the load is defined as the velocity, which in an undamped case induces infinite displacements directed upward as well as downward. Trains moving with a velocity that is getting close to the critical one, induce excessive vibrations on the supporting structure, which consequently interacts with the train. This increases supporting structure deterioration and negatively affects passengers comfort and neighbouring structures. If the critical velocity is reached, more serious faults can occur, like train derailment, fatigue failure of the rails or power supply disruption [1].

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Since a considerable amount of studies have been published on this subject, only a few pioneering works are mentioned here. Dynamic stresses in the beam structure were first solved by Krylov [2] and later by Timoshenko [3]. Transverse vibrations in a simply supported beam traversed by a constant force moving at a constant velocity were presented by Inglish [4], Lowan [5] and, later on, other solutions have been given by Koloušek [6] and Frýba [7]. In these approaches, the deflection field is expressed as an infinite sum of normal modes. Each mode contribution can be obtained by methods of integral transforms [8].

The first solution of a steady-state dynamic response of an infinite beam on an elastic foundation traversed by a moving load was presented by Timoshenko [9]. The Fourier transform was used for solving the ordinary differential equation. In [10], the effect of the foundation viscous damping on the response is discussed. The case of a load variable over time is presented in [11]. An important comparison between finite and infinite beam characteristics is presented in [12]. In [13,14], the concept of the dynamic stiffness matrix is implemented. Two semi-infinite beams are solved and connected by continuity equations. Then the critical velocity can be determined as the velocity that ensures the nullity of the determinant of the dynamic stiffness matrix.

It can be shown that in the steady-state regime, load exerts no inertial effects [7], which is probably the reason why also the mass inertia of the foundation was overlooked and the formula for the critical velocity of a load traversing an infinite beam supported by Winkler's foundation was used for many years in railway transportation:

$$v_{cr}^{E-B} = \sqrt[4]{\frac{4kEI}{m^2}} \tag{1}$$

In Eq. (1), v_{cr} is the critical velocity, E–B designates that Euler–Bernoulli theory is used; k, EI and m stand for the foundation stiffness (Winkler's constant), bending stiffness and mass per unit length of the beam. Considering the application in railway transportation, for a standard 60E1 rail it holds approximately $EI = 6.4$ MN m^2 and $m = 60$ kg m^{-1}. Values of k documented in the literature cover an interval from 0.22 MN m^{-2} to 1000 MN m^{-2} [13,15], thus the lowest value of the critical velocity is still above 700 km h^{-1}, which is not what has been observed in reality, especially on soft soils [1,16].

Formula (1) is closely related to finite beams, which is important to mention in view of further derivations. Following [17], a resonant velocity of a load moving on a finite beam corresponds to the velocity, for which the excitation frequency of the passing load is equal to the corresponding beam natural frequency. Such a resonant velocity can be attributed to each vibration mode. The critical velocity is the lowest resonant velocity. For a simply supported Euler–Bernoulli beam of length L on Winkler's foundation, the resonant velocity is

$$v_{res,j}^{E-B} = \frac{L}{j\pi}\sqrt{\left(\frac{j\pi}{L}\right)^4 \frac{EI}{m} + \frac{k}{m}} \tag{2}$$

and its minimum value is achieved for j_{cr}, that is the closest integer to $\tilde{j}_{cr}$, determined from the stationary condition as

$$\tilde{j}_{cr} = \frac{L}{\pi}\sqrt[4]{\frac{k}{EI}} \tag{3}$$

Then the critical velocity, given by Eq. (2) with j_{cr} substituted, is always higher than the one from Eq. (1) related to infinite beams. But using the real number $\tilde{j}_{cr}$ in Eq. (2), Eq. (1) is confirmed. This method can be extended to other situations.

Several studies have been performed over the years in order to generalise Eq. (1). There are two possible directions of doing it: by generalisations of the beam and/or of the foundation models. As far as the beam is concerned, by nullity condition imposed on the determinant of the dynamic stiffness matrix, or by the method described above, the value for the Timoshenko–Rayleigh (superscript "T–R") beam can be obtained as [17]:

$$v_{cr}^{T-R} = \sqrt{\frac{k\left(EI\left(kr^2 - G\overline{A}\right) - 2\left(rG\overline{A}\right)^2\right) + 2G\overline{A}\sqrt{kG\overline{A}}\sqrt{kr^4 G\overline{A} - EI\left(kr^2 - G\overline{A}\right)}}{m\left(kr^2 - G\overline{A}\right)^2}} \tag{4}$$

where r, $\overline{A}$ and G stand for the radius of gyration of the beam cross-section, the reduced (by Timoshenko's shear coefficient) cross-sectional area and the shear modulus of the beam, respectively. In case of railway applications, when the beam is modelled by the rail, such an extension does not bring much alteration. It can, however, be useful for equivalent beams formed by the track, ballast and embankment, as in [1,16,18].

Regarding the foundation model, two different directions have been followed. In one of them, the foundation was replaced by an elastic half-space. In such a model it has been shown that the critical velocity of the moving load corresponds to the velocity of propagation of Rayleigh waves in the foundation [19]. Experimental evidence of such critical velocity is reported in [1,16]. In [18,20], it has been concluded that the problem is more complicated, and besides the Rayleigh-wave velocity, there is a critical velocity resulting from the dynamic interaction between the beam and the elastic half-space.

Another direction related to the foundation model generalisations, suggested improvements of the Winkler model [21] by introduction of another parameter in so-called Filonenko–Borodich or Pasternak models [22]. This parameter is introduced to account for the coupling effect of the Winkler linear elastic springs, it represents the shear contribution and can equally be understood as distributed rotational spring. The model is named as a "two-parameter model". Further, in order to