Journal of Sound and Vibration 366 (2016) 447–470

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Kinematic and dynamic modeling and approximate analysis of a roller chain drive

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ARTICLE INFO

Article history:
Received 6 October 2014
Received in revised form
30 November 2015
Accepted 14 December 2015
Handling Editor: L.N. Virgin
Available online 5 January 2016

Keywords:
Roller chain drive
Axially moving string
Moving boundaries
Polygonal action
Kinematic analysis
Multibody simulation

ABSTRACT

A simple roller chain drive consisting of two sprockets connected by tight chain spans is investigated. First, a kinematic model is presented which include both spans and sprockets. An approach for calculating the chain wrapping length is presented, which also allows for the exact calculation of sprocket center positions for a given chain length. The kinematic analysis demonstrates that the total length of the chain wrapped around the sprockets generally varies during one tooth period. Analytical predictions for the wrapping length are compared to multibody simulation results and show very good agreement. It is thereby demonstrated that chain drives with tight chain spans must include compliant components to function. Second, a dynamic model is presented which includes the two spans and the driven sprocket. Assuming the presence of a stationary operating state, the presented dynamic model allows for analytical studies of the coupled motion of the chain spans and driven sprocket. Parametric excitation of the spans come from sprocket angular displacements, and the driven sprocket acts as a boundary which can be compliant in the axial direction. External transverse excitation of the spans comes from polygonal action, and is treated through kinematic forcing at the moving string boundaries. Perturbation analysis of the model is carried out using the method of multiple scales. Results show a multitude of internal and external resonance conditions, and some examples are presented of both decoupled and coupled motion. Together, the kinematic and dynamic model are aimed toward providing a framework for conducting and understanding both numerical, and experimental investigations of roller chain drive dynamics.

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1. Introduction

Roller chain drives are applied for power transmission in many mechanical systems due to a high energy efficiency, large power capacities, timing capabilities, flexibility in choosing shaft center distance, and ease of installation and maintenance. However, roller chain drives are also challenging due to the presence of undesired noise and vibration, and is therefore subject to ongoing studies [1].

Kinematic studies of roller chain drives are carried out by modeling the sprockets as polygons [2]. The angular motion of two sprockets connected by a chain span is considered to happen through a series of four-bar mechanisms [3]. Because a chain wrapped around a sprocket forms a polygon rather than a circle, several less desirable effects are introduced. These are

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http://dx.doi.org/10.1016/j.jsv.2015.12.028
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Nomenclature

Latin

$a(T_1)$	real-valued slow modulation amplitude for transverse vibrations
A	cross section area of string
A_j–D_j	chain drive configurations, $j=1,2,3$ (def. in Table 1)
$A(T_1)$	complex-valued slow modulation amplitude for transverse vibrations
c_u, c_w	wave speed of transverse and axial string waves, resp.
C	driver sprocket
C_{mn}	modal expansion coefficient (def. in Eq. (59))
d	$\hat{d}_1$ non-dimensionalized by $R_1^2\sqrt{\rho AP_t}$
$\hat{d}_1$	rotational viscous damping coefficient for driven sprocket
$D_j^k(\,)$	k'th partial derivative with respect to T_j
e	axial strain with mean value subtracted
e_0	mean axial strain
E	Young's modulus of string material
E_m	modal expansion coefficient (def. in Eq. (59))
f	pitch fraction
f_2	rotation frequency (in Hz) of driver sprocket
$f_m(t)$	m'th modal forcing component
$\hat{f}_1$	brake load
G_m	modal expansion coefficient (def. in Eq. (59))
h	binary function (def. in Eq. (2))
i	imaginary unit
J	$\hat{J}_1$ non-dimensionalized by ρAlR_1^2
$\hat{J}_1$	mass moment of inertia of driven sprocket and machinery, resp.
k_p	Fourier coefficient
K	kinetic energy of string
l	driven to driver sprocket center distance
l_a	length of upper (tight) span
l_b	length of lower (slack) span
L	total chain length
$L_{\max}$	maximum wrapping length
L_{mean}	mean wrapping length
$L_{\min}$	minimum wrapping length
m	mode number
M	total number of chain links
M_1	output torque non-dimensionalized by R_1P_t
M_2	input torque non-dimensionalized by R_2P_t
$\hat{M}_1, \hat{M}_2$	nominal (mean) output and input torque, resp.
$\hat{M}_1^*, \hat{M}_2^*$	time varying part of output and input torque, resp.
n	sprocket tooth number
n_1, n_2	number of teeth on driven and driver sprocket, resp.
$\hat{n}$	number of seating curves with a roller seated
$\check{n}$	number of seating curves with no roller seated
N	in Sections 2–4: number of chain links in a chain span; In Sections 5–6: number of modes included in Galerkin expansion
$N([x,]t)$	instantaneous string tension
O	driven sprocket
$O(\,)$	order of magnitude
p	chain pitch length
p_0, p_1, p_2	non-dimensional harmonic forcing amplitudes for the single-mode approximation
p_0^*	additional tension from time-varying wrapping length
P	actual span tension
P_0	mean steady-state span tension
P_{pre}	span pretension
P_t	reference span tension (for non-dimensionalizing time)
P_{tot}	total pretension (two spans)
q_0, q_1	multiple scales expansion functions for $\xi_m(t)$
Q_1, Q_2	acceleration jump at driven and driver sprocket, resp.
r	radius of pitch polygon inscribed circle
R	radius of pitch circle
R_1, R_2	radius of driven and driver pitch circle, resp.
s	S non-dimensionalized by $\sqrt{P_t/\rho A}$
S	nominal span velocity
t	time non-dimensionalized by $\sqrt{\rho Al^2/P_t}$
t_u, t_l	upper and lower tangent of inscribed sprocket circles
T	time
T_0, T_1	slow and fast time in multiple scales analysis
u	U non-dimensionalized by l
u_1, u_2	U_1, U_2 non-dimensionalized by l
u_2^*	U_2^* non-dimensionalized by l
U	axial displacement of chain or string point
U_1, U_2	axial displacement (wrt. X_1, X_2) of moving string endpoints
U_2^*	prescribed/kinematically forced value of U_2
$v(T_1)$	real-valued slow modulation amplitude for rotational vibrations of driven sprocket
V	potential (elastic) energy of string
$V(T_1)$	complex-valued slow modulation amplitude for rotational vibrations of driven sprocket
w	non-dimensionalized transverse chain or string displacement with rigid body mode subtracted
$\hat{w}$	$\hat{W}$ non-dimensionalized by l
$\hat{W}$	transverse displacement of chain or string point
x, y	X, Y non-dimensionalized by l
x_1, x_2	X_1, X_2 non-dimensionalized by l
X, Y	inertial coordinate system for upper chain span
X_1, X_2	left and right endpoint coordinates along X of moving string
X', Y'	inertial coordinate system for lower chain span
y_1, y_2	Y_1, Y_2 non-dimensionalized by l
Y_1, Y_2	transverse displacement (at X_1, X_2) of moving string endpoints

Greek

α	In Section 2.2: pitch angle; elsewhere: axial string stiffness EA non-dimensionalized
α_1, α_2	pitch angle of driven and driver sprocket, resp.
α_m	modal expansion coefficient (def. in Eq. (61))
β	angle btw. coord. systems (X, Y) and (X', Y')
γ	P_0 non-dimensionalized by EA
$\delta(\,)$	variation
$\Delta x_1, \Delta x_2$	small axial variations of string support positions

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