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Horizontally shaken impact pendulums

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ABSTRACT

We consider two pendulum masses attached to the same pivot point and which interact with each other through Hertzian impacts. We show that this splitting of the mass leads to an instability in the conservative case, in which initially synchronized large amplitude motion may evolve into out-of-phase (impacting) motion. We then study in detail the response of the impacting masses in the presence of damping and driving through horizontal shaking of the pivot point. We find that synchronized modes are usually accompanied by small amplitude quasi-periodic, or even chaotic, impacts and a number of multi-period solutions may appear in the bifurcation diagram. We reveal the existence and stability of a number of impact modes and scan the frequency response of the system to a series of initial conditions to identify which modes may be more easily generated in experiment.

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1. Introduction

Impact dynamics are of broad interest due to their appearance in a wide range of physical systems [1]. In the context of engineering applications the examples are numerous and varied, including cases such as percussive drilling [2,3], gear tooth wear [4], and robotic locomotion [5]. Parallel to the engineering studies there has been significant interest in simpler physical models to uncover the new effects which emerge due to impact [6]. The appearance of grazing bifurcations was revealed in a horizontally shaken pendulum with a tilted impact plane (see, e.g. [7]). Newton's cradle continues to be studied, with dissipation shown to result in marked change from the canonical case [8]. Even the seemingly simple case of two bouncing balls has revealed that a finite elastic impact response can have a dramatic effect on the resulting dynamics [9]. In this work we continue this theme and explore the changes which occur when a single horizontally shaken pendulum mass (see e.g. [10]) is replaced with two masses attached to the same central pivot point (see Fig. 1(a)). In this way we can explore explicitly the effect of impacts in a simple driven pendulum system.

Pendulum-like configurations with impacts have received a significant amount of attention as they allow identification of the effects of impact in systems with a relatively small number of degrees of freedom. A widely studied case is that of a pendulum interacting with an external block placed at some angle (see e.g. [7]). This has recently been extended to the case of two impacting masses interacting with an external block [11]. We consider a similar configuration, except we take the two impacting pendulums to be identical, and with no external impacting block. In this way we are able to examine the new features which emerge relative to a single driven pendulum, when the mass is able to "split" into two. Earlier work has also considered a similar configuration [12], except this earlier work used a vertical pivot point oscillation (which makes fixed

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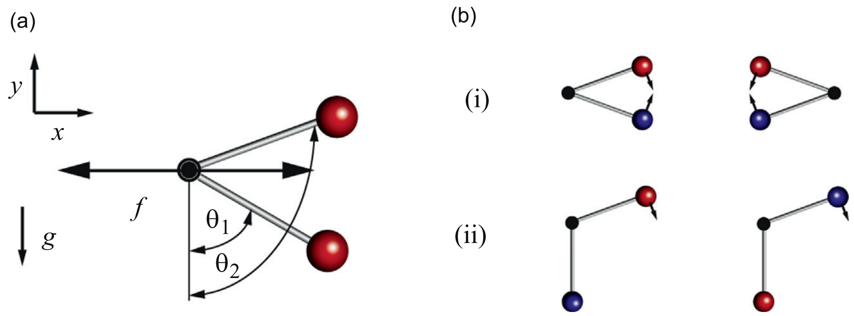

Fig. 1. (a) Schematic of two pendulums, interacting through impacts, and driven by a horizontally shaken pivot point. (b) Two examples of modes which emerge due to the impacts: (i) out-of-phase oscillation/rotation impact mode; (ii) ratchet-like rotation.

points possible) and two point-masses interacting instantaneously through impacts. We consider a more physical scenario, in which the masses have a finite size and interact through Hertzian restoring forces on impact.

We identify the changes which occur relative to a single pendulum system and reveal some of the modes which appear due to the impacts. We follow the families of these modes and identify their stability. We show that the presence of impacts leads to significant complexity in the single-pendulum-like modes, due to the possibility of the two masses undergoing small amplitude oscillations relative to each other when in contact. We find also modes which have no connection to the single-pendulum case and which emerge solely due to the increased number of degrees of freedom. Finally we move to a more physical consideration of the dynamics which may be observed in the system starting from equilibrium and from a static inverted configuration. By scanning the driving frequency we identify which modes are preferentially excited and we provide some general conclusions concerning the effect of driving strength and mass size on the dynamics. Overall we find that at low initial energy the system will try to mimic the single-pendulum dynamics, but that at higher initial energy impact modes may spontaneously appear.

The paper proceeds as follows. In Section 2 we present the model of our impact system, including a physical discussion of the parameters, and then rewrite the system in terms of new coordinates for the centre-of-mass and mass separation. In Section 3 we present examples of solutions which may be observed in the dynamics, beginning with discussion of the effects of impacts in the conservative case, and followed by a discussion of first the single pendulum case and then the full driven impact case. In Section 4 we follow the various modes in the driven impact case and explore in more detail some of the complexity surrounding the single-pendulum-like dynamics in the presence of impacts. In Section 5 we examine from a physical perspective the types of solutions which emerge from fixed initial conditions, specifically the cases of “zero” and “inverted” initial configurations. We consider a variety of cases, including weak and strong forcing, and the case of large, compliant masses. We finish in Section 6 with our general conclusions.

2. Model

We consider two identical masses of mass M attached to the same pivot point by a rigid rod of length R and mass m. The masses are constrained to lie in the xy plane, as shown in Fig. 1(a), with acceleration due to gravity g acting in the negative y direction. To be consistent with the standard pendulum literature, we define θ_i as the angle of the ith mass relative to the negative y-axis. The functions of the x and y positions of the ith mass are given by $x_i(t) = R\sin\theta_i + F\cos\omega t$ and $y_i(t) = R\cos\theta_i$ respectively, where we assume that the pivot oscillates harmonically about the origin along the x-axis. The masses are free to interact with each other through elastic impacts. We assume that upon impact the masses are free to deform, and that the restoring force is purely Hertzian in nature (see for instance [13] for a derivation of the elastic Hertzian contact theory).

We consider the following normalized equation of motion for the angle θ_i for the ith mass ($i = 1, 2$):

$$\ddot{\theta}_i + \gamma\dot{\theta}_i + \sin(\theta_i) - f\omega^2\cos(\omega t)\cos(\theta_i) + c(\theta_w - (\theta_{i+1} - \theta_i))^{3/2} - c(\theta_w - (\theta_i - \theta_{i-1}))^{3/2} = 0. \quad (1)$$

There is a natural periodicity to the system, such that $\theta_{N+1} \equiv \theta_1 + 2\pi$ and $\theta_0 \equiv \theta_N - 2\pi$ which allows us to use Eq. (1) when $N=2$. Parameter $\gamma = \Gamma/(I\omega_0)$ captures the on-site resistive damping (with physical damping coefficient Γ describing, for instance, damping at the pivot point), where $I = MR^2 + mR^2/3$ is the moment of inertia of a rotating mass and $\omega_0 = \sqrt{g/R}$ is the natural frequency of a single mass oscillating with small amplitude. The normalized forcing amplitude is given by $f = F/R$ and time has been normalized using the natural frequency ω_0. The normalized driving frequency $\omega = \omega_d/\omega_0$ is the ratio of the driving frequency ω_d to the natural frequency. The angular radius of a mass is given by $\theta_w/2$ where $\theta_w \lll 2\pi$ such that contact between neighbours can be assumed, for simplicity, to occur at a point collinear with the centres-of-mass. The impact coupling coefficient $c = C/(I\omega_0^2)$ is large and positive when neighbouring masses are in contact, and zero otherwise. Note that contact occurs whenever the argument of the coupling term is positive (i.e. whenever the 3/2 power terms are real). In this way Eq. (1) can continuously model the dynamics both during impact and when the masses are not in contact. We focus on the case of a hard mass, given by $c=50\,000$, and can quantify the meaning of this by calculating the impact depth for masses impacting with velocities $v = \pm v_i$. By equating kinetic energy with the Hertzian potential we see that the

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