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Local resonance and Bragg bandgaps in sandwich beams containing periodically inserted resonators

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ABSTRACT

We study the low frequency wave propagation behavior of sandwich beams containing periodically embedded internal resonators. A closed form expression for the propagation constant is obtained using a phased array approach and verified using finite element simulations. We show that local resonance and Bragg bandgaps coexist in such a system and that the width of both bandgaps is a function of resonator parameters as well as their periodicity. The interaction between the two bandgaps is studied by varying the local resonance frequency. We find that a single combined bandgap does not exist for this system and that the Bragg bandgaps transition into sub-wavelength bandgaps when the local resonance frequency is above their associated classical Bragg frequency.

1. Introduction

The creation of wave attenuation frequency bands due to addition of substructures to a host medium was first demonstrated in the context of acoustic waves by Liu et al. [1]. Various researchers have since studied the attenuation of acoustic and elastic waves by utilizing such locally resonating elements [2]. The localized resonance of the added substructures causes energy sequestering and prohibits the propagation of incident waves [3]. These local resonance bandgaps are distinct from those obtained due to multiple scattering and interference effects occurring between periodic elements in a propagation medium, classically referred to as Bragg bandgaps [4–6]. Due to the nature of the mechanism involved in their creation, Bragg bandgaps are generated at wavelengths comparable to the spatial scale of the periodicities [5]. Local resonance bandgaps, on the other hand, are independent of the spatial organization of the resonant substructures and are solely governed by the unit cell resonance frequency [7].

Recently, researchers have investigated various systems containing periodically placed local resonators and demonstrated the coexistence of both bandgaps in the same system [8–20]. Still et al. [8] experimentally demonstrated the existence of hypersonic Bragg as well as local resonance bandgaps in three dimensional colloidal films of nanospheres and also showed that for a structurally disordered system, the Bragg bandgap disappears while the local resonance bandgap persists. Croënne et al. [9] reported their coexistence as well as an overlap between the two bandgaps for a 2D crystal of nylon rods in water. Similarly, Achaoui et al. [10] showed the presence of local resonance bandgaps at low frequencies and Bragg bandgaps at high frequencies for surface guided waves in a lithium niobate substrate with nickel pillars, while

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Bretagne et al. [11] reported similar results for acoustic wave propagation through a 3D bubble phononic crystal. Kaina et al. [12], Chen et al. [13], and Yuan et al. [14] have recently reported results demonstrating coupling between the two bandgaps and creation of a single combined 'resonant-Bragg' bandgap extending over a wide frequency range. For flexural wave propagation, Liu et al. [15] utilized a transfer matrix method to study the effect of various periodicities and their combination on the bandgap characteristics of a beam. Specifically, for the case of suspended mass periodicity they showed that local resonance and Bragg bandgaps coexist in the system and also provided a transition criterion for the two bandgaps in terms of the group velocity gradient. Others have used a similar transfer matrix approach to study different beam structures with periodically placed resonators [16–18]. Xiao et al. used the plane wave expansion method [19] and a spectral element formulation [20] to obtain the propagation constant for an Euler–Bernoulli beam with attached local resonators, and demonstrated a super-wide combined pseudo gap when a resonance gap was nearly coupled with a Bragg bandgap.

The idea of the utilization of space available in thick sandwich cores to accommodate locally resonating elements was first proposed by Chen et al. [21]. The high stiffness to weight ratio of sandwich structures makes them an ideal solution to save energy through weight reduction. Consequently, sandwich beams have gradually found increased applications in the aerospace, marine as well as the automotive sectors [22]. However, a major roadblock to further adoption of composite sandwich designs is their susceptibility under dynamic loading [23]. Chen et al. [24] demonstrated that embedding resonators inside the sandwich core generates local resonance wave attenuation bandgaps which help improve their dynamic flexural performance without a significant weight penalty. To analyze the effect of resonators embedded inside the core, they assumed the resonators to be uniformly distributed over the entire length of the beam using a volume averaging technique. Doing so allows the description of such a resonator-embedded beam using continuous field variables and the equations of motion were derived using Hamilton's principle. However, this model did not account for discrete resonators inserted in the core with a spatial periodicity and was unable to capture the effect of such a periodicity on the dispersion behavior of the system.

The motivation for this study is to obtain a more complete understanding of the effect of periodically inserted resonators on the wave propagation behavior of sandwich beams. We adopt the phased array method to obtain the propagation constants for a sandwich beam with resonator embedded core. The phased array method was developed by Mead [25] to obtain closed form solutions for the propagation constants of various periodic systems. Since we are primarily interested in the low frequency behavior of the system, we model the sandwich beam as an equivalent Timoshenko beam [21,26] and treat the resonators as a phased array of forces. Though the presence of damping has been shown to alter the bandgap widths [2], here we assume an idealized system and ignore any possible damping effects. Dispersion curves obtained by this method are compared with volume averaging method and finite element results and it is shown that local resonance and Bragg bandgaps coexist. The relationship between bandgap bounding frequencies, resonator stiffness and mass, and the periodic distance is analyzed in the context of modal frequencies of simple unit cell models [6]. Finally, the interaction between local resonance and Bragg bandgaps is studied and the possibility of creating a single combined bandgap is considered.

2. Method

The conventional sandwich construction involves bonding two thin facesheets on either side of a thicker, lightweight core material. Typically, facesheets provide the bending rigidity while the shear stiffness is provided by the core.

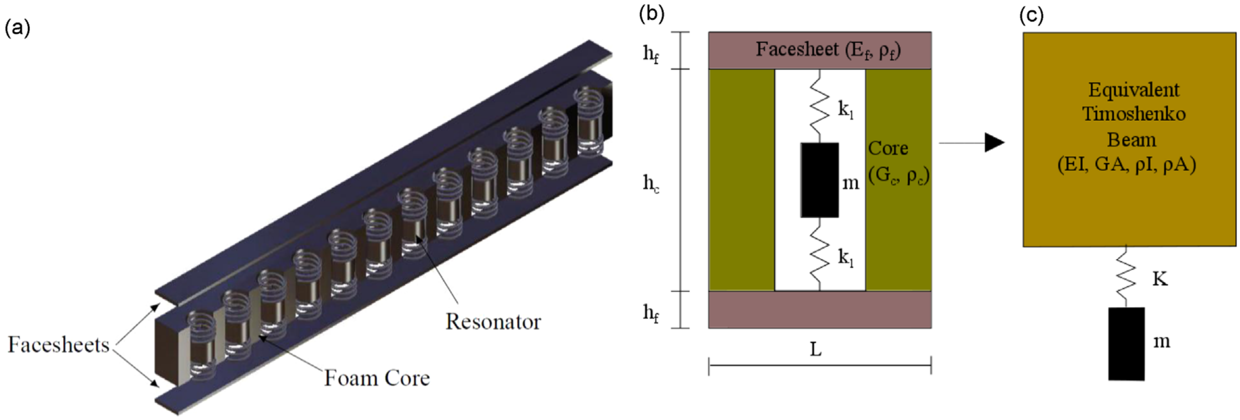

Fig. 1. (a) Schematic of a sandwich beam with internal resonators; (b) unit cell showing the relevant geometrical and material parameters of individual components; and (c) equivalent unit cell modeled as a Timoshenko beam with attached resonators. The equivalent bending rigidity EI, shear rigidity GA, mass per unit length ρA, and rotary inertia ρI are calculated using Eqs. (1)–(4).