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A reciprocal identity method for large silencer analysis

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ABSTRACT

Conventional techniques used in the boundary element method for evaluating muffler transmission loss have been limited by the cutoff frequency of the inlet and outlet ducts. Even though the boundary element method itself is a truly three-dimensional analysis tool, it has not been effectively used on large silencers due to the large inlet and outlet cross sections. In this paper, a numerical technique based on the reciprocal identity and the boundary element impedance matrix is proposed as a post-processing filter to extract the transmission loss of large silencers at all frequencies. Each reciprocal identity couples two different sound fields on the same silencer geometry. The first sound field has the analytical modal expansion in the inlet and outlet ducts, while the second sound field is the boundary element solution associated with a random boundary condition set. Depending on how many modes exist in the inlet and outlet ducts at a certain frequency, a minimum number of random boundary condition sets must be applied to the boundary element model. The boundary element impedance matrix provides more than enough such solution sets for the reciprocal identity coupling. The overdetermined system is then solved by a least-squares procedure. The proposed method is verified by comparing to the analytical solutions of a simple expansion chamber and a round bar silencer.

1. Introduction

Mufflers and silencers are devices to attenuate exhaust noise in various environments. Reactive mufflers take advantage of an impedance mismatch due to geometry changes (i.e., area contraction and expansion, side branch resonators). An impedance mismatch will prompt a backward-traveling wave reflected from the junction. Dissipative mufflers, on the other hand, use sound absorbing materials to convert sound energy to heat. In practice, many mufflers use both mechanisms since reactive elements effectively abate sound at low frequencies, while dissipative mufflers are better suited to addressing high-frequency broadband noise [1].

The most commonly used metrics for evaluating the acoustical performance of mufflers are insertion loss (IL), transmission loss (TL), and noise reduction (NR) [2,3]. Transmission loss is often the first step of analysis since it represents the inherent capability of the muffler to attenuate sound if both the source and termination are assumed to be anechoic. The TL is defined as the difference in dB between the incident sound power upstream and the transmitted sound power downstream [3]. In practice, if a muffler is already built and available, the TL below the plane-wave cutoff of the inlet and outlet can be measured in the lab by using the two-source method [4] or the two-load method [5]. Both methods actually measure the four-pole transfer matrix of the muffler, and the TL is computed from the four-pole matrix afterwards. On the other hand, if a muffler is still in the design stage, either a one-dimensional analysis tool based on the four-pole transfer matrix theory or a three-dimensional numerical

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method, such as the boundary element method (BEM) or the finite element method (FEM), is used to predict the TL. It is well known that a one-dimensional analysis tool is good up to the plane-wave cutoff frequency of the muffler chamber. Naturally, it is limited to low frequencies only. A three-dimensional method (BEM or FEM) can theoretically go to a much higher frequency. Kirby and his co-workers [6–10] used a hybrid FEM to study the acoustical performance of large dissipative silencers. To apply the hybrid technique, the 2D FEM is first employed to extract the eigenvalues and the associated eigenvectors of an axially uniform cross section. These 2D transversal modes are then used in the modal expansion along the axial direction if the cross section remains the same. To determine the unknown amplitudes in the modal expansion, either a point collocation method or a mode matching scheme is adopted to enforce the continuity of sound pressure and particle velocity at both ends where the uniform section meets the flanges or any irregular junctions. All the modes, including the evanescent modes, are considered in the modal expansion because the evanescent modes are still important at the flanges or irregular junctions. Since the FEM is mainly used on a 2D cross section to extract the modes, the hybrid FEM is a very efficient numerical technique for silencers with a very long axially uniform section. Due to the large cross sections at inlet and outlet for such applications, Mechel [11] suggested three different source models, constant modal amplitude, equal modal power, and equal modal energy density. The equal modal energy density source model is believed to be most realistic.

For BEM, however, the computation of TL has been limited by the plane-wave cutoff frequency of the inlet and outlet. This restriction is due to the fact that conventional TL evaluation methods for the BEM still rely on the plane-wave decomposition in the inlet and outlet ducts, even though the BEM itself is a truly three-dimensional modeling tool. The conventional techniques used to compute TL below the plane-wave cutoff of the inlet and outlet ducts include the three-point method [12] and the improved four-pole method [13]. The four-pole matrix does not exist above the plane-wave cutoff because sound pressure and particle velocity are not uniform at the inlet and outlet. More importantly, the anechoic termination can no longer be represented by the simple characteristic impedance boundary condition. To go beyond the plane-wave cutoff at the inlet and outlet, modal expansions must be introduced in the BEM. In fact, modal expansions have been used in both the Green's function and the sound pressure in the boundary integral equation itself for underwater waveguide problems [14,15]. Modal expansions have also been used in a hybrid BEM for ultrasound defect detection [16–20]. In the hybrid BEM, modal expansions in the far field (away from any defects) are coupled to the BEM in the near field at the junctions between these two fields. Like the hybrid FEM, all the modes must be considered in the expansion because evanescent modes are still important at junctions.

The direct mixed-body BEM [21,22] has been used to model mufflers and silencers with complex internal components, such as extended inlet/outlet tubes, thin baffles, flow plugs, perforated tubes, and multiple bulk-reacting materials. The main advantage of the BEM is that only the surface is meshed. Unlike the conventional BEM, which is limited to one homogeneous acoustic medium only, the direct mixed-body BEM can include different bulk-reacting materials along with air in one single BEM domain. For a very long silencer, a substructuring technique [23] can also be used to divide the silencer into several smaller substructures along the longitudinal direction. Each substructure can still contain complex internal components and multiple bulk-reacting materials, and can have its own temperature to partially account for the temperature gradient. The impedance matrix that relates the inlet to the outlet of each substructure is calculated by the BEM, and an impedance matrix synthesis procedure is then applied to combine all the individual impedance matrices into one resultant impedance matrix for the whole silencer. Below the plane-wave cutoff, the resultant impedance matrix can be further condensed into a 2×2 four-pole matrix in order to compute the TL. It should be noted that the 2×2 four-pole matrix no longer exists above the cutoff.

In this paper, a numerical technique based on the classical reciprocal identity is proposed to determine the TL of large silencers without mean flow at all frequencies. In general, the reciprocal identity is an integral equation that couples two different sound fields on the same muffler geometry. It can be proved that the reciprocal identity integral eventually reduces to the inlet and the outlet surfaces only. All other surface integrals over either the chamber surface or any internal component surfaces disappear. The first sound field to be used in the reciprocal identity coupling is an analytical solution with a given incident wave at the inlet and an anechoic termination at the outlet. This analytical solution represents an ideal condition under which the TL is defined. It is well known that the complete analytical solution for a complex real-world muffler could be very difficult to obtain. Fortunately, only the general modal expansion in the inlet and outlet ducts is needed because the reciprocal identity integral has been reduced to the inlet and outlet surfaces only. The second sound field to be used in the reciprocal identity coupling is the BEM solution associated with an arbitrary boundary condition. Like in the improved four-pole method [13], the BEM computation does not require an explicit anechoic termination. In fact, any random boundary condition set may be used in the BEM. Each reciprocal identity that couples the two sound fields, one analytical and the other BEM associated with a random boundary condition set, forms one linearly independent equation. Depending on how many modes are used in the modal expansion of the analytical solution at the inlet and outlet, there will be a finite number of unknown amplitudes associated with the reflected and transmitted waves. It should be noted that most evanescent modes won't survive at the inlet and the outlet of the silencer because one can always extend the inlet and outlet ducts without changing the TL. In fact, the decay rate of any evanescent mode along the inlet/outlet duct can be analytically assessed by examining the power in the exponential function of the evanescent mode. Normally, one or two evanescent modes are enough, although one can always choose to include more without adding too much computational cost. To solve for the unknown amplitudes, a minimum number of random boundary condition sets must be applied in the BEM. If the number of random boundary condition sets in the BEM exceeds the number of unknown amplitudes in the analytical expansion, the system is solved by a least-squares procedure.