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In-plane dynamic stiffness matrix for a free orthotropic plate

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ABSTRACT

The aim of this paper is to describe a procedure for computing the dynamic stiffness matrix relative to the in-plane effect for an orthotropic rectangular plate. The dynamic stiffness matrix is calculated for free edge boundary conditions. The formulation is based on strong solutions for the equations of motion for an orthotropic plate obtained with the Levy series and a Gorman decomposition of the free boundary conditions. The results obtained for the in-plane harmonic response are validated by the Finite Element Method.

1. Introduction

Structural vibrations are one of the main problems investigated by mechanical engineering in the domain of transport. Much research has focused on the vibration analysis of plates [1,2]. In-plane vibration is very important for understanding the energy transmission between two coupled plates [3–5] and modeling sandwich composite plates [6]. Bardell et al. [7] presented a significant contribution to understanding in-plane vibrations by using the Rayleigh–Ritz method to calculate the in-plane vibrational frequencies for plates with simply supported edges. Farag and Pan [8] expanded a solution for the in-plane vibration of a rectangular plate when the opposing edges are clamped. Yufei Zhang et al. [9] used a double Fourier cosine series to present a series solution for the in-plane vibration analysis of an orthotropic rectangular plate with elastically restrained edges and compared the analytic results with those obtained using the finite element method. Gorman extended the problem of lateral vibrations when analytical types of solutions are used for the in-plane vibration of rectangular plates [10], making it possible to apply the superposition method [11]. Gorman also presented the analytical method of in-plane solutions for the natural frequencies and for the mode shapes of simply supported and clamped rectangular plates [12]. In addition, Xing et al. [13] presented the exact solution for the in-plane natural frequencies of a rectangular plate when its opposing edges are simply supported. Du et al. [14] applied the Fourier series method to analyze the in-plane vibration of a rectangular plate with elastically restrained edges while Seok et al. [15] analyzed the free in-plane vibration for a rectangular cantilever plate. They used a variational method and an equation of motion for analyzing in-plane vibrations of thin orthotropic plates. Woodcock et al. [16] expanded the Hamilton principle and the Rayleigh–Ritz method to study the effects of the ply orientation on in-plane vibrations. Park [17] used the separation of the variables to derive the

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equations for the clamped circular plate. Moreover, Gorman presented the exact solution for a rectangular plate when its two opposing edges are simply supported and the others are clamped or free [18].

The Dynamic Stiffness Method has proved efficiency for analyzing the harmonic response for complex structures composed of simple elements [19–21]. This meshless method characterized by the absence of structural discretization is based on exact solutions of the harmonic motion equations for free boundary conditions. The method uses the dynamic stiffness matrix $\mathbf{K}(\omega)$ that links the boundary displacements denoted $\mathbf{U}$ and the external forces applied on the boundaries denoted $\mathbf{F}$, according to

$$\mathbf{K}(\omega)\cdot\mathbf{U}=\mathbf{F} \tag{1}$$

Dynamic stiffness matrices have been developed for many elements. Casimir et al. [22] and Banerjee et al. [23,24] developed the dynamic stiffness matrix of various beams, while Tounsi et al. studied circular rings [25]. Boscolo and Banerjee [26] focused on in-plane vibration of isotropic plates according to the assumption that two opposite sides are simply supported. Some researchers have also investigated the dynamic stiffness matrix of shells [27–29]. The main objective of this paper is to develop the dynamic stiffness matrix of an orthotropic plate for in-plane vibrations according to the assumption that all the edges of the plate are free. Natural boundary conditions for the four edges are required to deal with future assemblies. In order to achieve this formulation, we used Gorman decompositions of four symmetry contributions and Levy type solutions, as explained in a previous paper concerning flexural vibrations [30]. A such approach has been recently used by Nefovska-Danilovic and Petronijevic for the problem of in-plane vibrations of isotropic plates [31]. The dynamic stiffness matrix $\mathbf{K}(\omega)$ is built by superposing four symmetry contributions. Following this, the validation of the formulation is achieved by comparisons of harmonic responses obtained by the Finite Element Method.

2. In-plane orthotropic plate equations

2.1. Internal forces–displacements relationship

Let us consider an orthotropic rectangular plate characterized by its lateral dimensions $2a$ and $2b$, and its thickness h. It is assumed that the principal material axes are parallel with the edges of the plate and with the axes of the Cartesian coordinate system denoted Ox and Oy. The in-plane displacements of any point on the middle surface of the plate along the x-axis and the y-axis are denoted u and v, respectively (see Fig. 1). The relationship of the stress/displacements is given by the following equations:

$$\begin{cases} \sigma_x = D_x\dfrac{\partial u}{\partial x}+D_1\dfrac{\partial v}{\partial y} \\ \sigma_y = D_1\dfrac{\partial u}{\partial x}+D_y\dfrac{\partial v}{\partial y} \\ \sigma_{xy} = D_{xy}\left(\dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y}\right) \end{cases} \tag{2}$$

where σ_x, σ_y and σ_{xy} are the stress tensor components. D_x, D_y, D_1 and D_{xy} are the material constants defined by the following equation:

$$\begin{cases} D_x = \dfrac{E_x}{1-\nu_{xy}\nu_{yx}} \\ D_1 = \dfrac{\nu_{xy}E_y}{1-\nu_{xy}\nu_{yx}} = \dfrac{\nu_{yx}E_x}{1-\nu_{xy}\nu_{yx}} \\ D_y = \dfrac{E_y}{1-\nu_{xy}\nu_{yx}} \\ D_{xy} = G_{xy} \end{cases} \tag{3}$$

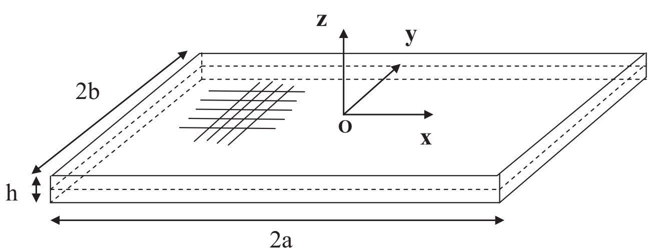

Fig. 1. Orthotropic plate.