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Nonlinear vibrations of viscoelastic rectangular plates

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ABSTRACT

Nonlinear vibrations of viscoelastic thin rectangular plates subjected to normal harmonic excitation in the spectral neighborhood of the lowest resonances are investigated. The von Kármán nonlinear strain–displacement relationships are used and geometric imperfections are taken into account. The material is modeled as a Kelvin–Voigt viscoelastic solid by retaining all the nonlinear terms. The discretized nonlinear equations of motion are studied by using the arclength continuation and collocation method. Numerical results are obtained for the fundamental mode of a simply supported square plate with immovable edges by using models with 16 and 22 degrees of freedom and investigating solution convergence. Comparison to viscous damping and the effect of neglecting nonlinear viscoelastic damping terms are shown. The change of the frequency–response with the retardation time parameter is also investigated as well as the effect of geometric imperfections.

1. Introduction

Plates made of soft materials, like rubbers and biomaterials, present viscoelastic characteristics of dissipation, e.g. delayed strains as a consequence of the stress application. Also viscoelastic materials and viscoelastic layers are used to increase damping of structures.

Vibration of viscoelastic plates has received attention in the literature (see e.g. [1]). But when it comes to geometrically nonlinear vibrations of viscoelastic plates, the literature is quite scarce. The Kelvin–Voigt viscoelastic model was used by Xia and Lukaziewicz [2,3] for modeling free and forced nonlinear vibrations of sandwich rectangular plates with simply-supported moveable edges. They treated the two external layers as elastic and the core as viscoelastic. The numerical solution was obtained by direct integration of the equations of motion by Runge–Kutta method.

The dynamic buckling and chaos of viscoelastic plates with nonlinear strain–displacement relationship has been studied by Sun and Zhang [4]. The plate is modeled as a standard linear solid-type viscoelastic material which gives rise to a single-degree-of-freedom integro-differential dynamic equation.

Rossihkin and Shitikova [5] used the Riemann–Liouville fractional derivative of order smaller than one to describe viscoelasticity. Nonlinear free damped vibrations of a rectangular plate discretized by three nonlinear ordinary differential equations were considered when the plate is being under the conditions of one-to-one internal resonance and the internal additive or difference combination resonances.

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Bilasse et al. [6] studied linear and nonlinear vibrations of a three-layer rectangular plate with different boundary conditions by neglecting in-plane and rotary inertia. The central core is considered to be viscoelastic and modeled as a Kelvin–Voigt material. Numerical results are given for certain loss factors, or for frequency dependent material properties. Effects of different core materials, as well as temperature, have been investigated. A finite element approach has been applied in a method that couples the harmonic balance technique to the complex mode Galerkin's procedure.

A harmonic balance method for the nonlinear vibration of viscoelastic rings and shells has been introduced by Boutyour et al. [7].

Mahmoudkhani and Haddadpour [8] investigated nonlinear vibrations of viscoelastic sandwich plates under narrow-band random excitations. The viscoelasticity is described by the single-integral nonlinear viscoelastic model. The system is discretized by using seven nonlinear integro-partial differential equations. Mahmoudkhani et al. [9] studied the nonlinear free and forced bending vibration of sandwich plates with an incompressible viscoelastic core described by the single-integral nonlinear viscoelastic model. The method of multiple scales (5th-order) is directly applied to solve the equations of motion.

Balkan and Mecitoğlu [10] approach the problem of the nonlinear dynamics of composite sandwich plates under blast load. The viscoelasticity is described by the Kelvin–Voigt model and clamped edges are considered. Only the core is considered viscoelastic and to bear shear, which is modeled by using the first-order shear deformation theory with von Kármán's nonlinear terms. The solution is obtained by the Galerkin method and direct time integration by discretizing the system with five degrees of freedom. Numerical and experimental data are compared.

Here the literature review of nonlinear vibration of elastic plates (not viscoelastic) is not reported but it can be found in Chia [11], Sathyamoorthy [12] and Chia [13]; curved panels and shells were reviewed by Amabili and Païdoussis [14] and Alijani and Amabili [15]. Nonlinear vibrations of rectangular plates have been studied by Ribeiro and Petyt [16,17] by the finite element method. Comparison of numerical and experimental results for nonlinear vibrations of thin rectangular plates with viscous damping is given by Amabili [18,19]. Nonlinear vibration of sandwich composite plates with viscous damping has been investigated numerically and experimentally by Alijani and Amabili [20] and Alijani et al. [21]. Alijani and Amabili [22] recently developed an 8-parameter theory to study nonlinear vibrations of laminated composite rectangular plates retaining nonlinearities in rotations and thickness deformation; in this study 60 degrees of freedom are used to discretize the equations of motion that retain all the geometric nonlinear terms in addition to the higher-order shear and thickness deformation, in-plane and rotary inertia.

In the present study, nonlinear vibrations of viscoelastic thin rectangular plates subjected to normal harmonic excitation in the spectral neighborhood of the lowest resonances are investigated. The von Kármán nonlinear strain–displacement relationships are used and geometric imperfections are taken into account. The material is modeled as a Kelvin–Voigt viscoelastic solid by retaining all the nonlinear terms. The discretized nonlinear equations of motion are studied by using the arclength continuation and collocation method. Numerical results are obtained for the fundamental mode of a simply supported square plate with immovable edges by using models with 16 and 22 degrees of freedom and investigating the solution convergence. Comparison to viscous damping and the effect of neglecting nonlinear viscoelastic damping terms are shown. The change of the frequency–response with the retardation time parameter is also investigated, as well as the effect of geometric imperfections.

2. The Kelvin–Voigt viscoelastic model and the Duffing equation

Before studying the nonlinear vibrations of viscoelastic rectangular plates, it is useful to see the effect of viscoelasticity on a simpler model. For this reason, in this section viscoelasticity is analyzed on a single degree of freedom model, before introducing it on the continuous model of the plate that is introduced in the next section. The simplest model for nonlinear vibrations of a plate or shell is given by the extended Duffing equation, which is the forced mass-spring system with viscous damping, where the restoring force of the spring is nonlinear with quadratic and cubic terms. Since the original Duffing equation contains just cubic nonlinear terms, here the term extended Duffing equation is introduced to indicate that quadratic nonlinearities are also included.

It is possible to generalize the extended Duffing equation and to substitute the viscous dissipation with viscoelasticity. In particular, we make use of the Kelvin–Voigt viscoelastic material model [23] shown in Fig. 1, which give the following relationship for the restoring force F of the viscoelastic spring:

$$F = k(x)x + \mu \frac{\partial}{\partial t}\left[k(x)\, x\right], \tag{1}$$

where the spring stiffness $k(x)$ is a nonlinear function of the vibration displacement (for a plate it could the maximum displacement, e.g. at the center) x, t is time and μ is a viscoelastic coefficient that is measured in seconds and takes the name of retardation time [23]. The retardation time multiplied by the excitation frequency (in Hz) gives the loss factor. The Kelvin–Voigt model consists of a spring and dashpot in parallel so that they experience the same strain, and the total stress is the sum of the stresses in each element. This model cannot describe the relaxation response of the material, but it is effective for