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Experimental validation of theoretical methods to estimate the energy radiated by elastic waves during an impact

Maxime Farin [a,*], Anne Mangeney [a,b], Julien de Rosny [c], Renaud Toussaint [d], Jacques Sainte-Marie [b], Nikolaï M. Shapiro [a]

[a] Institut de Physique du Globe de Paris, Sorbonne Paris Cité, CNRS (UMR 7154), France
[b] ANGE team, CEREMA, Inria, Lab. J.-L. Lions, CNRS, France
[c] ESPCI, Institut Langevin, CNRS, 1 rue Jussieu, 75005, Paris, France
[d] Institut de Physique du Globe de Strasbourg, Université de Strasbourg/EOST, CNRS, 5 rue Descartes, F-67084 Strasbourg Cedex, France

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ABSTRACT

Estimating the energy lost in elastic waves during an impact is an important problem in seismology and in industry. We propose three complementary methods to estimate the elastic energy radiated by bead impacts on thin plates and thick blocks from the generated vibration. The first two methods are based on the direct wave front and are shown to be equivalent. The third method makes use of the diffuse regime. These methods are tested for laboratory experiments of impacts and are shown to give the same results, with error bars of 40 percent and 300 percent for impacts on a smooth plate and on a rough block, respectively. We show that these methods are relevant to establish the energy budget of an impact. On plates of glass and PMMA, the radiated elastic energy increases from 2 percent to almost 100 percent of the total energy lost as the bead diameter approaches the plate thickness. The rest of the lost energy is dissipated by viscoelasticity. For beads larger than the plate thickness, plastic deformation occurs and reduces the amount of energy radiated in the form of elastic waves. On a concrete block, the energy dissipation during the impact is principally inelastic because only 0.2–2 percent of the energy lost by the bead is transported by elastic waves. The radiated elastic energy estimated with the presented methods is quantitatively validated by Hertz's model of elastic impact.

1. Introduction

The quantification of the energy emitted by a source in the form of elastic waves is a common problem in various fields such as vibroacoustics or shielding. In seismology, the problem was confronted long ago [1] and many approaches have since been developed to estimate the energy of natural sources such as earthquakes (see [2–5]), tremors [6], landslides and rockfalls (e.g. [7–11]). In the literature, the power spectral density (PSD) of the emitted signal is often measured to quantify the relative energy of different acoustic sources located at the same distance from the sensor and to compare their frequency content. For example, the temporal evolution of the PSD can provide information on river discharge and on the grain size of the bed load (e.g. [12]). The PSD can also be used to characterize crack formation in brittle [13,14] or granular materials (see [15], for review) and other crackling or crumpling processes (e.g. [16,17]). Finally, acoustic measurements can

* Corresponding author. Tel.: +33 1 83 95 77 82.
E-mail address: farin@ipgp.fr (M. Farin).

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Nomenclature

B	bending stiffness (J)
c_P, c_S, c_R	longitudinal, shear and Rayleigh wave speeds (m s^{-1})
e	coefficient of restitution (–)
e_{tot}, e_c, e_p	bulk densities of total, kinetic and potential energies (J m^{-3})
$\tilde{e}_{\text{tot}}$, $\tilde{e}_c$, $\tilde{e}_p$	time Fourier transform of e_{tot}, e_c, e_p, respectively (J m^{-2})
E	Young's modulus (Pa)
E_c, ΔE_c	energy of the impact and energy lost during the impact (J)
$E_{\text{tot}}(t)$	total elastic energy radiated within the structure at time t (J)
f	frequency (s^{-1})
$\tilde{G}^P_{zz}$, $\tilde{G}^S_{zz}$, $\tilde{G}^R_{zz}$	vertical Green's functions associated with compressional, shear and Rayleigh waves (kg^{-1} s^2)
h	plate thickness (m)
k	wavenumber (m^{-1})
L, S, V	length (m), surface area (m^2) and volume (m^3)
m	bead mass (kg)
r, θ, z	coordinates in the cylindrical reference frame (m)
S_{ij}	strain tensor (–)
t	time (s)
T_{ij}	stress tensor (Pa)
$\mathbf{u}_i$	normalized vector of direction i
u_i, v_i, a_i	surface displacement, speed and acceleration in the direction $\vec{u_i}$ (m; m s^{-1}; m s^{-2})
$\tilde{U}_i$, $\tilde{V}_i$, $\tilde{A}_i$	time Fourier transform of u_i, v_i and a_i, respectively (m s; m; m s^{-1})
v_g, v_ϕ	group and phase velocities (m s^{-1})
V_z	speed of a bead before impact (m s^{-1})
W_{el}, $W^{\text{th}}_{\text{el}}$	radiated energy and theoretical radiated energy (J)
x, y, z	coordinates in the Cartesian reference frame (m)
β, ξ	parameters involved in energy calculations
γ	attenuation coefficient of energy with distance (m^{-1})
λ, μ	Lamé coefficients of compression and shear (Pa)
ν	Poisson's coefficient (–)
π_P, π_S, π_R	energy partitions among P, SV and Rayleigh waves (–)
π^{surf}_P, π^{surf}_S, π^{surf}_R	surface energy partitions among compressional, shear and Rayleigh waves (–)
$\tilde{\Pi}$	energy density flux (J m^{-1} s)
ρ	density (kg m^3)
τ	characteristic time of energy attenuation (s)
χ, η	viscoelastic coefficients of compression and shear (Pa s)
ω	angular frequency (s^{-1})

be useful in industry for particle sizing in powder transport and in particle streams (e.g. [18,19]). However, the PSD does not provide an absolute estimate of the elastic energy radiated by the source because it depends on the distance of measurement.

There are three main approaches to determine the absolute radiated elastic energy from acoustic emissions. The first method consists in computing the energy flux crossing a surface surrounding the source. The integration of the energy flux over this surface gives the radiated power. This technique is applied in seismology to estimate the energy radiated in elastic waves during earthquakes (e.g. [5,20]) and rockfalls (e.g. [8–10]).

The second technique to deduce the radiated elastic energy is based on the estimation of the time dependence of the source force. Miller and Pursey [21] and Goyder and White [22] thus estimated the power radiated in an elastic half-space and in an infinite plate, respectively, by a monochromatic harmonic force. In most cases, the force profile is generally unknown but it can be retrieved from the deconvolution of the displacement field with Green's function tensor [3].

These two first methods can however be performed only when the emitted wave front is not mixed with its reflections off the boundaries of the elastic solid. If multiple side reflections occur, the transported energy becomes homogeneously distributed within the elastic solid and decreases exponentially with time due to viscoelastic dissipation. This situation is commonly referred to as a diffuse field in the literature (see [23–25]). A third energy estimation method, called the diffuse method hereafter, thus consists in extrapolating the radiated energy at the instant of the source from the exponential decrease of the signal coda (see e.g. [25,26], and references therein).

The energy flux, deconvolution and diffuse field methods to estimate the energy lost in elastic waves are used separately by different communities and are based on different assumptions. The first two methods require a sufficiently large elastic solid so that the direct wave front can be clearly distinguished from its reflections off the lateral sides of the elastic solid. On the contrary, with the diffuse method, the elastic solid must be small enough so that multiple side reflections occur. To our knowledge, no study has ever compared these three methods in cases where all three can be applied.

The complex seismic signals generated by rockfalls, bed load transport in rivers and granular flows are partially composed of waves generated by the collisions of individual impactors (gravels, boulders, etc.). Therefore, if we hope to understand these signals, we must first understand the energy budget of individual impacts. The energy that is not radiated in elastic waves during an impact is lost by plastic deformation i.e., not reversible, of the impactor or of the surface [27], by local viscoelastic dissipation around the contact [28] and by conversion into other degrees of freedom of the impactor's motion, such as rotation and other displacement modes. Because of the significant differences between the conditions of each impact on the field, it is however not clear how the energy budget of the impactor depends on its size and speed.