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On sound scattering by rigid edges and wedges in a flow, with applications to high-lift device aeroacoustics

Michel Roger [a,1], Stéphane Moreau [b,*,2], Korcan Kucukcoskun [c,3]

[a] *École Centrale de Lyon, 69134 Écully, France*
[b] *Université de Sherbrooke, Département de génie mécanique, Sherbrooke, QC, Canada J1K2R1*
[c] *École Centrale de Lyon, 69134 Écully, France*

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ABSTRACT

Exact analytical solutions for the scattering of sound by the edge of a rigid half-plane and by a rigid corner in the presence of a uniform flow are considered in this work, for arbitrary source and observer locations. Exact Green's functions for the Helmholtz equation are first reviewed and implemented in a quiescent propagation space from reference expressions of the literature. The effect of uniform fluid motion is introduced in a second step and the properties of the field are discussed for point dipoles and quadrupoles. The asymptotic regime of a source close to the scattering edge/wedge and of an observer far from it in terms of acoustic wavelengths is derived in both cases. Its validity limits are assessed by comparing with the exact solutions. Typically the asymptotic directivity is imposed by Green's function but not by the source itself. This behaviour is associated with a strong enhancement of the radiation with respect to what the source would produce in free field. The amplification depends on the geometry, on the source type and on the source distance to the edge/wedge. Various applications in aeroacoustics of wall-bounded flows are addressed, more specifically dealing with high-lift device noise mechanisms, such as trailing-edge or flap side-edge noise. The asymptotic developments are used to highlight trends that are believed to play a role in airframe noise.

1. Introduction

The role of solid surfaces in aeroacoustics is twofold. Firstly they participate in the generation of sound by direct interaction with flows. Secondly they redistribute the sound radiated by sources possibly located elsewhere. In that sense they act as either sources of sound or scattering obstacles and can be artificially classified as active or passive surfaces, respectively. The physical understanding and the modeling of both aspects are key issues in the definition of noise reduction strategies in many engineering applications. Pure sound scattering is usually investigated using the classical theory of diffraction in linear acoustics. Aerodynamic sound production in the presence of solid surfaces can be formulated from the standpoint of linear acoustics by resorting to the acoustic analogy. According to Lighthill's original statement [1] and related

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[*] Corresponding author.
E-mail addresses: michel.roger@ec-lyon.fr (M. Roger), stephane.moreau@usherbrooke.ca (S. Moreau).
[1] Laboratoire de M'ecanique des Fluides et Acoustique.
[2] D´epartement de G´enie M´ecanique.
[3] Current position in Siemens Industry Software.

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Nomenclature

Italic symbols

A_0	Kutta-correction factor
c	chord length
c_0	speed of sound
D	directivity factor
E	Fresnel integral
F	complex function for the 2D half-plane Green's function
$\mathbf{F}$	dipole strength vector
G	Green's functions for the Helmholtz equation
G_K	correction to Green's function for the Kutta condition
G_{M_0}	Green's functions with uniform flow
G_∞	2D asymptotic Green's function in free field
h	flap side-edge thickness
H	Heaviside function
$J_\nu, H_\nu^{(1)}$	Bessel and Hankel functions of the first kind
$k = \omega/c_0$	acoustic wavenumber
$K = k/\beta$	modified wavenumber
k_1^*	dimensionless aerodynamic wavenumber
K_n	modified Bessel functions
ℓ	unsteady lift on a flap
$l_{\max}$	size of quadrilateral elements
M_0	Mach number
p	acoustic pressure
P	dipole strength
P_ν^μ	general Legendre functions
q	monopole strength
$\mathbf{Q}$	quadrupole strength tensor
$(\overline{r}, \overline{\theta}, z)$	corrected observer cylindrical coordinates
$(\overline{r}_0, \overline{\theta}_0, z_0)$	corrected source cylindrical coordinates
$r_> = \max(\hat{r}, \hat{r}_0)$	maximum distance
$r_< = \min(\hat{r}, \hat{r}_0)$	minimum distance
$\overline{r}$, $\overline{r}_{1,2}$, $\overline{r}_{1,2}$	corrected 3D and 2D scattering distances
$S_{1,2}$, $s_{1,2}$	3D and 2D integral bounds in convected Green's functions
S_0	convection-corrected distance
$T_{i,j}$	quadrupole strength components
U_0	flow speed
$\mathbf{x} = (x, y, z)$	observer Cartesian coordinates for an edge
$\mathbf{x}_0 = (x_0, y_0, z_0)$	source Cartesian coordinates for an edge
$\mathbf{x} = (r, \theta, z)$	cylindrical observer coordinates for an edge
$\mathbf{x}_0 = (r_0, \theta_0, z_0)$	cylindrical source coordinates for an edge
$\mathbf{x} = (x_1, x_2, x_3)$	observer Cartesian coordinates for a corner
$\mathbf{x}_0 = (y_1, y_2, y_3)$	source Cartesian coordinates for a corner
$\mathbf{x} = (\hat{r}, \Theta, \phi)$	observer spherical coordinates for a corner
$\mathbf{x}_0 = (\hat{r}_0, \Theta_0, \phi_0)$	source spherical coordinates for a corner
$X_0, X = x_0/\beta, x/\beta$	modified streamwise coordinates for an edge
$X_1, Y_1 = x_1/\beta, y_1/\beta$	modified streamwise coordinates for a corner

Greek symbols

α_d	dipole inclination angle
$\beta = \sqrt{1 - M_0^2}$	compressibility parameter
ε_m	constant in Green's function for rigid wedge
$\overline{\theta}_1$	angle in the correction to Green's function for the Kutta condition
$\overline{\theta}_0, \overline{\theta}$	corrected spherical source and observer angles
λ	acoustic wavelength
ϕ^{2D}, ϕ^{3D}	two-or three-dimensional acoustic potentials
Φ	wedge aperture angle
φ	projection angle for the half-plane Green's function
ρ_0	fluid density
ω	angular frequency

Subscripts/superscripts symbols

κ, m	summation indices
K	Kutta-condition correction
M_0	flow-corrected quantity
$(1/2)$	half-plane

works by Howe [2] and Ffowcs Williams and Hall [3], for instance, unsteady flow patterns interacting with solid surfaces can be interpreted as equivalent quadrupoles distributed in the fluid, the direct sound of which is scattered by the surfaces. This view is developed in some asymptotic theories of high-lift device noise [4,5]. According to Ffowcs Williams and Hawkings' statement of the analogy [6], surfaces explicitly involved in noise generation can also be mathematically interpreted as equivalent sources of lower orders. *A priori* the latter point of view is well suited for active surfaces and the former for passive surfaces. But the distinction is questionable when two bodies in close vicinity of each other are embedded in a disturbed flow region. The present analysis is dedicated to the high-lift devices that are deployed when the wing of an aircraft operates in approach and landing conditions. The source and scattering surfaces are implicitly assumed to be well separated, which only covers a part of the complete physics in most cases of interest. For instance a deployed flap (Fig. 1) can be interpreted as a distribution of equivalent sources, the sound of which is scattered by the main part of the wing. The underlying mechanism partly contributes to the airframe noise that also includes landing-gear associated sources. Airframe noise in itself is recognized as a major contributor to the total noise of an aircraft at approach, essentially because the modern high by-pass ratio engines are much quieter at idle power.

Going into the details, High-Lift Device (HLD) noise involves distributed sources along the leading or trailing edges of the wing ((2) in Fig. 1), the slat and the flap ((1) and (4) in Fig. 1), as well as sources that concentrate around the span ends of wing elements such as flap side-edges ((3) in Fig. 1) or slat corners. In the present study two contributions in which sound