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Instability of asymmetric shaft system

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ABSTRACT

In the present work, parametric instability of asymmetric shaft mounted on bearings is studied. Towards this end, four different models of increasing complexity are studied. The equations corresponding to these models are formulated in the inertial reference frame. These equations involve a periodically varying coefficient. This is similar to classical Mathieu equation but in a multi-degree of freedom context. As such, under suitable parameter combination these systems result in growing oscillation amplitudes or instability. For wider generalization, the equations and results are presented in a non-dimensional form. The unstable parameter regimes are found using the Floquet theory and perturbation methods. These results are also corroborated with existing results in the literature. The nature of the stability boundary and its dependence on various system parameters is discussed in elaborate detail. The stability boundary can be used to determine unstable operating speed ranges for different asymmetric shaft cross-sections. Further, material, geometry and bearing selection guidelines for ensuring stable operations can be inferred from these results.

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1. Introduction

Parametric excitation in rotor dynamic systems frequently arises in different applications and is known to be a cause of their failure. Such systems are characterized by a time-varying parameter. Certain regimes of system parameters are known to cause instability resulting in large growth of vibration amplitudes. It is imperative for the designer to know *a priori* the parameter ranges for the stable and unstable operation to ensure the reliability of the system. In the present work, we investigate the instability due to such parametric excitation in a system comprising an asymmetric shaft mounted on symmetric bearings. Such asymmetric shafts frequently arise practical applications as keyed or splined shafts. The varying moments of inertia along different directions result in continuously changing stiffness of the rotating shaft, which eventually leads to parametric excitation and instability thereof.

Lee [1] in his works has presented a formulation for determining the stability or instability of rotating asymmetric shaft. The equations of motion for such shaft formulated in the inertial reference frame show a parametric effect due to changing stiffness. On transformation of the equations of motion to a reference frame fixed to the rotating shaft, the equations of motion do not bear any time-varying parameter and are simplified to a constant coefficient differential equation. Through these equations the unstable speed range for a given shaft cross-section is identified. However, in this work the authors limit their analysis to a lumped two degree of freedom model of the shaft system. Arnold and Haft [2] have extended this approach for continuous flexural deformation of the shaft. However, only transverse inertia and bending deformation are

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Nomenclature

γ	a number between 0 and 1
δK	difference in stiffness of the asymmetric shaft
δK^*	non-dimensional difference in stiffness of the asym- metric shaft
ζ	damping factor
ω	angular velocity
b	minor axis
$C_1, C_2\ldots$	arbitrary constants
C	equivalent viscous damping
C_m	monodromy matrix
E	Young's modulus of shaft material
f^*	non-dimensional displacement given by $\frac{f}{l}$
g^*	non-dimensional displacement given by $\frac{g}{l}$
G^*	non-dimensional gyroscopic parameter
h	major axis, key way hight
I_1, I_2	area moment of inertia evaluated about principal coordinates of an asymmetric shaft cross section
I_{yy}	area moment of inertia of asymmetric shaft cross section about y-axis
I_{zz}	area moment of inertia of asymmetric shaft cross section about z-axis
K	equivalent mean stiffness of the asymmetric shaft
K^*	non-dimensional mean stiffness of the asymmetric shaft
K_b	bearing stiffness
K_b^*	non-dimensional bearing stiffness
l	length of shaft
m	equivalent mass of the shaft
m_b	bearing mass
r^*	$\left\{\begin{matrix} y^* \\ z^* \end{matrix}\right\}$
r	$\left\{\begin{matrix} y \\ z \end{matrix}\right\}$
R	radius
t	time
u	displacement y transformed into rotating coordinates
v	displacement z transformed into rotating coordinates
y, y_1	shaft displacement along y-axis
y^*, z^*	$\frac{y}{l}$, $\frac{z}{l}$
y_1^*	non-dimensional displacement of shaft along y-axis
y_2^*	non-dimensional displacement of bearing along y-axis
z, z_1	shaft displacement along z-axis
z_1^*	non-dimensional displacement of shaft along z-axis
z_2^*	non-dimensional displacement of bearing along z-axis
*	represents non-dimensional quantities
$\ddot{}$	represents acceleration
$\dot{}$	represents velocity

accounted for in their formulations. Both analytical and experimental results of an overhung asymmetric shaft with asymmetric disc at the end are presented. System gyroscopy is neglected in the study. Frohrib [3] studied the stability of a rotor–shaft–bearing system accounting for gyroscopic effects. They evaluated the influence of shaft stiffness on instability. The coupling effect between the bearing and the asymmetric shaft has been studied by Rajalingham et al. [4]. They investigated the influence of bearing damping on stability using a lumped model with four degrees of freedom. The work is useful to suitably chose bearing parameter so that the instability can be avoided. Kang et al. [5–7] presented a more complete analysis with the inclusion of bending deformation, lateral inertia, rotary inertia, and gyroscopic inertia for stability. However, both internal damping and shear deformation are ignored. In contrast, Genta [8] proposed a finite element based solution for the equations of motion for a general rotating system. These equations of motion, likewise are formulated in a rotating reference frame. Further, the rotating system can include shafts, rotors, bearings or any combinations of them. The model is capable of handling viscous, hysteretic damping, rotating and non-rotating part asymmetry, unbalance distribution.

Frulla [9] used the Floquet theory to investigate the stability characteristics of a lumped bearing–shaft–rotor model. Ganesan [10] obtained analytical expressions of asymmetric shaft response and phase. The author emphasizes on near resonance behaviour of asymmetric shaft. Stability criteria are obtained based on effect of shaft asymmetry, and bearing damping. Pei [11] has studied an asymmetric shaft system with axial loading and gyroscopic effect. Results of two solution techniques namely Bolotin's method and the Floquet theory are compared. Author's main emphasis is on the assumptions of Bolotin's method which results in enlarged stability boundary that contradict with Floquet theory results. Raffa and Vatta [12] have studied a symmetric Bernoulli beam with shear effect and the Rayleigh beam with parametrically varying axial force. The assumed solution to the governing equation with simply supported shaft boundary conditions results in classical SDOF Matheiu equation. The stability of shaft with respect to parametrically varying axial force is described over stability boundary of Mathieu equation reported in the literature. Bartylla [13] has studied asymmetric shaft with parametrically varying axial loading. The Floquet theory is used to obtain the stability plot with respect to the parameters of parametrically varying axial force.

Parametric vibrations are also observed in rotating machinery with faults such as crack and geared rotor bearing system. Significant work is done in this area in which the authors studied the equation of motion in inertial frame. Shudeifat [14] used time varying moment of inertia to model cracked rotor in inertial frame. The formulated equation of motion in finite elements contains time varying stiffness matrix. The Harmonic Balance method is used to solve the governing equations. A

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