



Behavior of plane waves propagating through a temperature-inhomogeneous region

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ABSTRACT

Description and analysis of acoustic waves in ducts with a region containing temperature-inhomogeneous fluid represent a significant problem of scientific and practical interest. This interest is induced by the need of understanding how temperature fields affect acoustic processes which would lead to a more efficient design and control of systems involving thermoacoustic interactions. Most of the works addressing these problems limit themselves to the assumption of weak temperature profile gradients or to temperature profiles which do not connect neighboring temperature-homogeneous regions smoothly. In our work we investigate the behavior of plane acoustic waves that enter a region with an arbitrary temperature gradient. A polynomial character of the used temperature profile ensures smooth connection with constant-temperature regions. The one-dimensional wave equation for ducts with an axial mean temperature gradient is solved analytically. The derived solutions based on Heun functions extend the class of published exact analytical solutions of model wave equations taking into account the medium temperature gradient. Due to the property that our proposed polynomial temperature function has derivatives equal to zero at points which are connected with the surrounding temperature-homogeneous regions we can form more complex smooth temperature profiles for which it is possible to use the transfer matrix method.

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1. Introduction

Propagation of plane acoustic waves in ducts with axial variations of temperature is an issue of scientific and practical interest. To understand the behavior of plane acoustic waves which propagate through a temperature inhomogeneity, it is important to have at disposal exact analytical solutions of the corresponding model equations. This enables us to improve a number of devices where acoustic waves interact with non-uniform temperature fields, for instance automobile exhaust systems, combustors, thermoacoustic devices, etc. [1,2].

There have not been many works published addressing propagation of acoustic waves through a temperature-inhomogeneous region of fluid, particularly as far as analytical solutions are concerned.

In some of the published works their authors restrict themselves to the condition that temperature gradients in temperature-inhomogeneous region are sufficiently small, which enables them to employ perturbation methods to obtain approximate solutions, see e.g. [3–5]. Another group of the published works overcomes the above-mentioned restriction

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and their authors present exact solutions of the model equations for various temperature profiles (especially polynomial ones), whose steepness is arbitrary, see e.g. [1,6–9]. Authors of the work [10] proposed a method which consists in transforming the corresponding wave equation into an integral Volterra equation of the second kind including boundary layer losses. However, this method leads to a solution in an implicit form. Holzinger et al. [11] presented an exact solution for the acoustic wave propagation in a narrow duct with a polynomial temperature profile where the thermal and viscous boundary layer is considered as well. Nevertheless, there is only a relatively small class of temperature profiles, for which exact analytical solutions are known. Our work extends this class of temperature functions (profiles) by a polynomial temperature function that allows to model both temperature transient regions and regions with temperature barriers. In addition, in contrast to already published works the considered temperature function is smoothly connected with surrounding temperature-homogeneous regions thus, the derivatives of the temperature function equal to zero at the points of connections. It means that in many cases this temperature-function is physically more realistic.

As it is shown in our work, the model equation for the presented polynomial temperature function can be rewritten into the canonical form of Heun's equation. We can notice in recent times that this equation has attracted the attention of an increasing number of scientists and we can encounter it in various fields of physics, for example quantum mechanics, theoretical mechanics, dislocation theory, the theory of black holes, lattice systems in statistical mechanics, etc., see e.g. [12–14]. Within this work, we demonstrate that this equation finds its application in physical acoustics as well.

In what follows, the model equation and the polynomial temperature function are presented in Section 2. Heun's equation and its general solution are described in Section 3.1. Within Section 3.2 the particular solution for the polynomial temperature function is shown. Section 4 deals with an application of the found solutions for calculation of transmission and reflection coefficients. A plane wave propagating through the temperature-inhomogeneous region is investigated in Section 5. Due to the property that our proposed polynomial temperature function has derivatives equal to zero at points which are connected with the neighboring temperature-homogeneous regions we can form more complex smooth temperature profiles for which it is possible to use the transfer matrix method. This method is described in Section 6. The achieved results are summarized in Section 7.

2. Model equation and a polynomial temperature function

We assume linear acoustic propagation through a perfect, inviscid and non-heat-conducting gas which fills a constant-cross-sectioned duct with an axial temperature distribution. Considering a zero mean gas flow and plane harmonic acoustic waves propagating through this duct we can derive the following model equation (see e.g. [1])

$$\frac{\partial^2 p'}{\partial x^2} + \frac{1}{T(x)} \frac{dT(x)}{dx} \frac{\partial p'}{\partial x} - \frac{1}{c(x)^2} \frac{\partial^2 p'}{\partial t^2} = 0, \quad (1)$$

where p' is an acoustic pressure, $T(x)$ is a temperature function representing a temperature distribution and $c(x)$ is a small-signal sound speed which depends on the temperature function as

$$c(x) = \sqrt{\gamma RT(x)}, \quad (2)$$

where γ is the ratio of specific heats (adiabatic index), R is the specific gas constant which is given by the molar gas constant, divided by the molar mass of the gas.

To solve Eq. (1) it is convenient to rewrite it into the dimensionless form

$$\frac{\partial^2 \Pi}{\partial \sigma^2} + \frac{1}{\eta(\sigma)} \frac{d\eta(\sigma)}{d\sigma} \frac{\partial \Pi}{\partial \sigma} - \frac{1}{C^2(\sigma)} \frac{\partial^2 \Pi}{\partial \theta^2} = 0, \quad (3)$$

with

$$\Pi = \frac{p'}{\rho(T_c)c^2(T_c)} = \frac{p'}{\gamma p_0}, \quad \sigma = \frac{x}{L}, \quad \theta = \omega t, \quad \eta = \frac{T}{T_c}, \quad C^2(\sigma) = \frac{c^2(\sigma)}{\omega^2 L^2} = \frac{\gamma RT(\sigma)}{\omega^2 L^2} = \frac{\eta(\sigma)}{h^2}, \quad (4)$$

where T_c is a characteristic temperature, $c(T_c) = \sqrt{\gamma RT_c}$, $\rho(T_c) = p_0/(RT_c)$, p_0 is the atmospheric pressure, L is a characteristic length, ω is an angular frequency and $h = \omega L/c(T_c)$.

We assume that a plane wave impinges normally on a region with varying (mean) temperature of the width W in a waveguide separating two regions of different constant (mean) temperatures T_A and T_B , see Fig. 1. For the temperature-inhomogeneous region we assume a quartic polynomial temperature function. The polynomial temperature function is given by the following formula:

$$T(\sigma) = T_c \eta(\sigma) = T_c [1 + 2q_1(\sigma - s)^2 - q_2(\sigma - s)^4], \quad (5)$$

where q_1 , q_2 are optional parameters and s is a control parameter.

Assuming $q_1 = q_2 = q$ then the temperature function (5) is smooth and its derivatives equal to zero at the points ($x=0$, $x=W$) where the temperature-inhomogeneous region is attached to the temperature-homogeneous regions.

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