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Internal resonance of nonlinear sloshing in rectangular liquid tanks subjected to obliquely horizontal excitation

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ABSTRACT

Nonlinear sloshing in rectangular tanks subjected to obliquely horizontal, harmonic excitation is investigated when the internal resonance condition 1:1 is satisfied between the natural frequencies of predominate modes (1, 0) and (0, 2). Galerkin's method is employed to derive the nonlinear modal equations of motion for sloshing, considering nine sloshing modes. Then, van der Pol's method is applied in order to obtain the expressions of the frequency response curves for amplitudes and phase angles of the predominate modes. The frequency response curves are calculated and reveal that (0, 2) mode may occur even though it is not directly excited because it is nonlinearly coupled with (1, 0) mode due to the autoparametric terms. In the numerical simulations, it is found that planar motions of (1, 0) mode, clockwise and counter-clockwise swirl motions, and translational motions may appear. Furthermore, Hopf bifurcation occurs, and amplitude modulated motions (AMMs), including chaotic motions, may appear depending on the value of the excitation frequency. Three-dimensional distribution charts of the maximum liquid surface elevation are calculated to show the risk of liquid overspill. The influence of the difference between the horizontal excitation direction and the tank side on the frequency response curves is also examined. Bifurcation sets are calculated to clarify this influence. Experimental data confirmed the validity of the theoretical results.

1. Introduction

Sloshing dynamics is one of the most important issues in mechanical, civil, marine and aeronautical engineering. Sloshing sometimes causes damage to oil-storage tanks and serious overflow of contaminated water from spent fuel pools when earthquakes occur. In order to resolve these issues, many researchers have been engaged in theoretically analyzing sloshing behavior and conducting experiments. It is well known that sloshing exhibits nonlinear behavior at large amplitudes that causes complex phenomena. Therefore, nonlinear analysis of sloshing should be conducted in order to obtain accurate results for large amplitude sloshing. Pioneering theoretical research on nonlinear sloshing dynamics was conducted by Moiseev [1], Hutton [2], and Abramson [3] from the late 1950s to the 1960s. Numerous papers on sloshing have since been published and Refs. [4–6] include copious examples of such papers.

Liquid tanks have various shapes and cross-sections including rectangular, circular, square, and elliptical. Tanks with rectangular cross-sections are widely used for practical purposes in real systems. Examples include tanks in oil tankers and spent fuel pools. The analysis of sloshing in rectangular tanks is more fundamental compared to that for tanks with more

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complicated shapes. The governing equation of fluid motion was derived, and the perturbation method was applied to determine the natural frequencies and liquid surface elevations of free oscillations [7]. As a result, the natural frequency of sloshing depended on the sloshing amplitude and it changed from soft to hard spring nonlinear characteristics when the ratio of the liquid level and tank length was $1.07/\pi = 0.34$. Although Fultz obtained a different critical value of 0.28 in experiments [8], Hermann and Timokha explained the discrepancy between the theoretical and experimental critical values in [9]. Chester [10] conducted a theoretical analysis of sloshing in a rectangular tank using the two-dimensional, Navier–Stokes equations, and compared the numerical results with experimental data [11]. However, to facilitate the analysis, the velocity potential can be used instead of the Navier–Stokes equations. In order to employ the velocity potential, the liquid must be restricted to an inviscid, incompressible, and irrotational fluid. The velocity potential has been commonly applied to the governing equations for liquid motion in order to obtain the expressions for frequency response curves, and many papers have shown the theoretical results for sloshing in rectangular tanks [12–17]. The theoretical results have also been compared with experimental data [14,16,17].

It is well known that there are infinite natural frequencies for liquid sloshing. When these natural frequencies satisfy internal resonance conditions in liquid tanks, multiple sloshing modes may simultaneously appear. Because these modes are coupled with each other, complicated phenomena may occur. The transition from standing waves to traveling waves may occur in a rectangular tank subjected to horizontal, harmonic excitation due to mode interaction, and this was observed in experiments including time histories [18]. Three-dimensional waves may appear in rectangular tanks due to internal resonance, and they were examined by deriving the modal equations of motion for dominant sloshing modes and showing their time histories when a tank was subjected to pitching excitation [19]. In addition, the frequency response curves for three-dimensional waves were theoretically calculated and compared with experimental results [20].

Three-dimensional waves are more likely to appear in square tanks because internal resonance conditions are often satisfied. For example, 1:1 internal resonance may occur for the two lowest sloshing modes, (1, 0) and (0, 1), and Yoshimatsu and Funakoshi [21] theoretically investigated the nonlinear coupling between these two modes. They showed the influence of the excitation direction on the frequency response curves and demonstrated that clockwise (CW) or counter-clockwise (CCW) swirl motions, Hopf bifurcations, and chaotic sloshing appeared. Faltinsen et al. [22–24] conducted a series of theoretical analyses of three-dimensional sloshing which were compared with experimental results. They examined the influences of higher sloshing modes and the aspect ratio of the tank cross-section. In addition, they classified steady-state resonant waves into planar, diagonal and swirling sloshing motions [25]. Ikeda et al. [26] showed the frequency response curves for liquid elevations in square and nearly square tanks subjected to obliquely horizontal excitation. They demonstrated the appearances of planar, diagonal, swirling, and amplitude modulated motions (AMMs) including chaotic motions, and confirmed these theoretical results by experiments. Pilipchuk [27] theoretically investigated the energy exchange between (1, 0) and (0, 1) sloshing modes using the Hamiltonian system derived from the modal equations proposed in [26]. However, there are few papers on three-dimensional sloshing motions concerning internal resonances in rectangular tanks.

The present paper investigates sloshing in rectangular tanks when (1, 0) and (0, 2) modes have an identical natural frequency, i.e., the internal resonance condition 1:1 is satisfied. The tank is subjected to obliquely horizontal, harmonic excitation. Galerkin's method is employed to derive the nonlinear modal equations of motion for sloshing, considering nine sloshing modes. Van der Pol's method [28] is then applied to those modal equations in order to obtain the expressions for the frequency response curves. In the numerical results, the influence of the angle of the horizontal excitation relative to the tank side on the frequency response curves is examined. Bifurcation sets are calculated to detail the influence of the excitation direction. The nonlinear interaction between (1, 0) and (0, 2) modes due to the autoparametric terms is clarified.

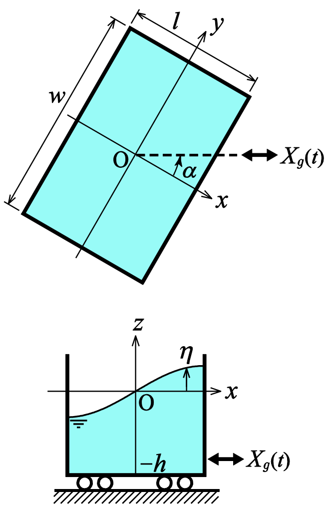

Fig. 1. The model for theoretical analysis.