



The speed of sound in periodic ducts



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ABSTRACT

The flow through corrugated pipes is known to lead to strong whistling tones which may be harmful in many industrial appliances. The mechanism is known to originate from a coupling between vortex shedding at the edges of the cavities forming the wall of the tube and the acoustical modes of the pipe. The latter depend upon the effective velocity of sound c_{eff} within the corrugated pipe. The purpose of this paper is to compute accurately this effective velocity of sound through an asymptotic calculation valid in the long-wave limit. Results are given for a number of geometries used in previous works, and compared with a simple model in which the effective speed of sound is function of the geometry of the pipe. The latter is found to work best for short cavities but significant disagreement is found for longer cavities. The case of 2D channels with a corrugated wall is also considered.

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1. Introduction

Corrugated pipes are commonly used in many engineering and industrial applications because they combine local rigidity with global flexibility. In many cases, the flow of air through such pipes can cause a loud whistling at specific frequencies. In most industrial cases, such whistling is an undesired effect as it may result in severe noise and structural vibration problems (see [1] for a recent review, covering industrial applications and available work on this problem). On the other hand, the ability of such ducts to generate clear and tonal sound is also used in a musical toy called the “hummer” or “Voice of the dragon” [2].

The mechanism responsible for whistling is known to combine an hydrodynamic and an acoustic subsystem [3,4]. The shear-layer instability, which is the hydrodynamic subsystem, acts as an amplifier and supplies acoustic energy to the system. On the other hand, the acoustic subsystem is represented by longitudinal standing waves, which act as a band-pass filter and maintain the synchronization in this feedback mechanism [5]. Note that the mechanism shares some similarity with the resonant cavity mechanism studied by Rossiter [6] for transonic flows except that the frequency is selected by the whole length of the tube, not the size of a single cavity. In this paper, only the acoustic part of the system is taken into account, then no mean flow is considered.

In a straight smooth tube of length L^* ¹ opened at both sides (where the definition of L^* may contain end corrections to account for matching with the outer regions), the acoustical frequencies are classically given by $f_m^* = m f_1^* = \frac{m c_0^*}{2L^*}$ ($m = 1, 2, 3, \dots$) where c_0^* is the speed of sound in unbounded medium. In the present case of a corrugated tube with periodic

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¹ In this paper all the dimensional quantities are indicated with the asterisk.

geometry, the presence of cavities at the walls results in an *effective speed of sound* c_{eff}^* which is lower than that in unbounded space. In the case where the mean flow corresponds to a low Mach number (which is effectively the case in most applications), the resonance frequencies are given by

$$f_m^* = m f_1^* = \frac{m c_{\text{eff}}^*}{2L^*}, \quad m = 1, 2, 3, \dots \quad (1)$$

so the knowledge of the *effective speed of sound* c_{eff}^* is thus crucial to predict the whistling frequencies of such devices.

Research on acoustic propagation has encompassed waveguides having various cross-sectional geometries, wall impedances and steady flow fields. For two-dimensional ducts, straightforward expansions of the form $\phi_0 + \epsilon \phi_1 + \dots$ were obtained by Isakovitch [7] for the case of a waveguide with only one sinusoidally undulating wall, by Samuels [8] for the case of a waveguide with in-phase wall undulations, and by Salant [9] for the general problem. In particular, in [9], he considers the propagation of acoustic waves through a plane waveguide consisting of two rigid sinusoidal walls by using a multiple scale approach. The same approach can be found in [10–12]. Nayfeh [10] discusses sound waves in two-dimensional ducts with sinusoidal walls. In [11], the same author extends this analysis to the case of linear waves propagating in a cylindrical hard walled duct whose wall has weak undulations which need not to be periodic. In [12], Nusayr uses the same method of multiple scales to analyze wave propagation in a rectangular hard-walled duct whose walls have weak periodic undulations.

Methods using a different approach have been suggested in the literature. In particular, in [13], Pagneux et al. present a multimodal expansion [14] to the problem of determining the wave propagation in waveguides with varying cross section. The same approach is used in [15,16], where a multimodal method is used to analyze the wave propagation in waveguides containing restrictions (or corrugations) with circular arc shapes. When the geometry of the duct is complex or it is not possible to derive a geometrical transformation to a simpler configuration, this approach may pose numerical difficulties in solving the equations, unlike the method of multiple scales. The propagation of time-harmonic acoustic waves in a cylindrical hard-walled duct has been also studied with the null field approach based on Green identities (see [17,18]).

Unlike what is done in [10–12], where the multiple scale approach is applied to geometries characterized by weak undulations of the wall whose amplitude is small compared with the height of the channel; here, there are no restrictions about the amplitude of the wall because the present formulation relies on the assumption that the acoustical wavelength is large compared with the characteristic dimension of the single corrugation, which is verified in most applications. Moreover, despite the multimodal method, it can be also applied to very complex configurations without consider any geometrical transformation.

The results obtained with our approach will be compared with a simple model, proposed by Cummings and initially published by Elliott [19], that gives an estimation of the effective speed of sound as a function of the geometry of the duct

$$\left(\frac{c_{\text{eff}}^*}{c_0^*} \right)^2 = \frac{V_{\text{in}}}{V_{\text{tot}}}, \quad (2)$$

where V_{in} is the volume of the inner part of the duct (excluding the cavities) and V_{tot} is the total volume (including the cavities). The argument leading to Eq. (2) relies on a quasi 1D modeling of Euler equations in which it is assumed that the velocity fluctuations do not vary across the pipe cross-section and the corrugations are modeled as small perturbation to the radial velocity. The mathematical derivation leading to Eq. (2) can also be found in [1], and a simpler derivation (in the case where the mean flow is neglected) is reproduced in the appendix of the present paper.

In Section 2, we detail the mathematical analysis and the numerical procedure. Then in Section 3 we provide results for a variety of geometries of tubes used in the literature, and compare with the simple estimate of Cummings–Elliott. Our analysis also works for 2D channels, and we briefly consider such geometries in Section 4. Then Section 5 summarizes the results and the open issues.

2. Mathematical analysis and numerical approach

The problem considered here is the propagation of acoustic waves in a corrugated pipe, which is assumed to be a periodic domain in the axial (x) direction. Fig. 1 depicts the typical geometry considered in the sequel: this description holds for both axisymmetric pipes (the lower boundary corresponding to the symmetry axis) and for two-dimensional channels (the lower boundary corresponding to a wall).

Let us consider an infinite pipe having a geometry like that shown in Fig. 1 as a fundamental building block. The numerical domain corresponds to a single periodic cell of pitch P_t^* of the corrugated pipe so the solution is imposed to be periodic along the length of the pipe. We describe the acoustic field through the velocity potential $\Phi(x, r)$, non-dimensionalized with the characteristic length of the pipe L^* and c_0^* , where $x = x^*/L^*$ and $r = r^*/L^*$ are respectively the streamwise and radial coordinates.

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