Journal of Sound and Vibration 361 (2016) 261–271

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

On sound propagation in a nozzle with non-uniform swirl

L.M.B.C. Campos *, P.G.T.A. Serrão

CCTAE (Center for Aeronautical and, Space Science and Technology), IDMEC/LAETA, Instituto Superior Técnico, Universidade de Lisboa, Av. Rovisco Pais 1, 1049-001 Lisboa, Portugal

ARTICLE INFO

Article history:
Received 6 February 2015
Received in revised form
22 July 2015
Accepted 1 September 2015
Handling Editor: Y. Auregan
Available online 21 October 2015

ABSTRACT

The propagation of sound is considered in an axisymmetric mean flow, with uniform axial velocity and non-uniform swirl, with angular velocity proportional to the radius. Two simplifications are made: (i) that the maximum tangential velocity of the mean flow is small compared to the sound speed; (ii) that the Doppler shift due to rotation is much smaller than the wave frequency (including Doppler shift by the uniform axial flow) divided by the maximum angular velocity of swirl. It is shown that these conditions are not too restrictive for practical swirling flows in exhaust nozzles, and that they allow the solution of the wave equation in terms of confluent hypergeometric functions (instead of Bessel functions in the case of rigid body rotation). This type of radial dependence allows for propagating waves with decaying amplitude in the case of 'slow' swirl, and for evanescent and unstable modes in the case of 'fast' swirl. The borderline between 'slow' and 'fast' swirl is given by a rotation parameter related to the second approximation (ii) above, and defined as the ratio of the Doppler shift due to rotation to the product of the frequency by the radial compactness.

1. Introduction

Swirling flows occur downstream of turbines and in the exhausts of jet engines and other turbomachinery [1–3]. Vorticity modes exist in an incompressible swirling flow [4,5] and couple to sound in the compressible case [6]. The simplest case is uniform or rigid body rotation [7], for which cut-off modes appear. The case of non-uniform rotation, with angular velocity decreasing with radial distance as a potential vortex, has been considered [8], including also the presence of uniform axial flow [9]. The present paper also includes uniform axial flow but considers non-uniform rotation, with angular velocity increasing proportionally to the radial distance.

The starting point is the acoustic wave equation in an axisymmetric mean flow, with arbitrary dependence on the radius of the axial velocity (shear flow) and angular velocity (swirling flow) [10]. The wave equation is simplified by the assumption (Section 2.1) of low Mach number shear and swirl, which in the case of uniform axial flow simplifies to the assumption of mean flow tangential velocity which is small compared to the sound of speed. The case of uniform or rigid body rotation is considered first (Section 2.1) to obtain the cut-off frequencies; the solution in terms of Bessel functions for rigid body rotation is replaced by a solution (Section 4.1) in terms of confluent hypergeometric functions, in the case of uniform rotation with angular velocity proportional to the radius.

The latter solution is obtained under a second assumption (Section 3.1): that the Doppler shift due to the rotation is small relative to the wave frequency (including Doppler shift due to the axial flow). It can be shown that the two assumptions are not too restrictive as regards typical jet engine cylindrical nozzles (Section 4.2), and they allow for the existence of

* Corresponding author.

http://dx.doi.org/10.1016/j.jsv.2015.09.001

(a) attenuated propagating waves for slow rotation and (b) monotonic evanescent and unstable modes for fast rotation (Section 3.2). The plots for typical exhaust nozzle conditions concern (Section 5.1) the amplitudes and phases of propagating modes for slow rotation (Figs. 1 and 2), and (Section 5.2) the waveforms of evanescent and unstable modes for fast rotation (Figs. 3 and 4). The borderline between the cases of slow and fast rotation is azimuthal wavenumber m times maximum angular velocity (at the wall) Ω_0 equal $m\Omega_0 = \vec{\omega}$ to the wave frequency ω with Doppler shift by the axial uniform flow $\vec{\omega} = \omega - kU$. Thus unstable modes occur for higher azimuthal order, faster rotation velocity and lower wave frequency (including Doppler shift by the axial mean flow).

There are other instances of the appearance of confluent or Gaussian hypergeometric functions in acoustic and related problems some of which are mentioned next. The first is quasi-one-dimensional sound propagation in an exponential nozzle containing a low Mach number mean flow whose velocity varies inversely with the cross-section to conserve the mass flux [11]. A second is the solution of the Tchapligyn equation [12] for the potential plane steady flow [13,14]. A third concerns acoustic-gravity waves in an atmosphere with an exponential temperature profile [15]. A fourth is vertical acoustic-gravity waves in an atmosphere in the presence of viscosity [16,17]. A fifth is non-dissipative magnetosonic gravity waves in an isothermal atmosphere in the presence of a uniform horizontal magnetic field [18,19]. A sixth is the propagation of Alfvén waves as transverse perturbations along magnetic field lines in an isothermal atmosphere in the presence of shear viscosity [20] or Ohmic resistivity [21] or both [22–24]. A seventh is Alfvén waves in an atmosphere with a non-uniform temperature [25]. An eighth magnetosonic waves in an atmosphere under a non-uniform magnetic field [26]. Several of the preceding cases are instances of the solution of a linear differential equation of any order whose coefficients are linear functions of an exponential in terms of confluent, Gaussian or generalized hypergeometric functions [27,28]. Historically or chronologically one of the first instances of the use of this property was the solution of the Rayleigh equation specifying the stability of an inviscid shear flow in the case of an exponential velocity profile [29,30].

2. Cylindrical nozzle with sheared and swirling mean flow

The acoustic wave equation in an axisymmetric mean flow with low Mach number shear and swirl (Section 2.1) is used to investigate the cut-off properties of sound in the simplest case of uniform axial flow and rigid body rotation (Section 2.2), before proceeding to cases (Sections 3–4) with non-uniform swirl.

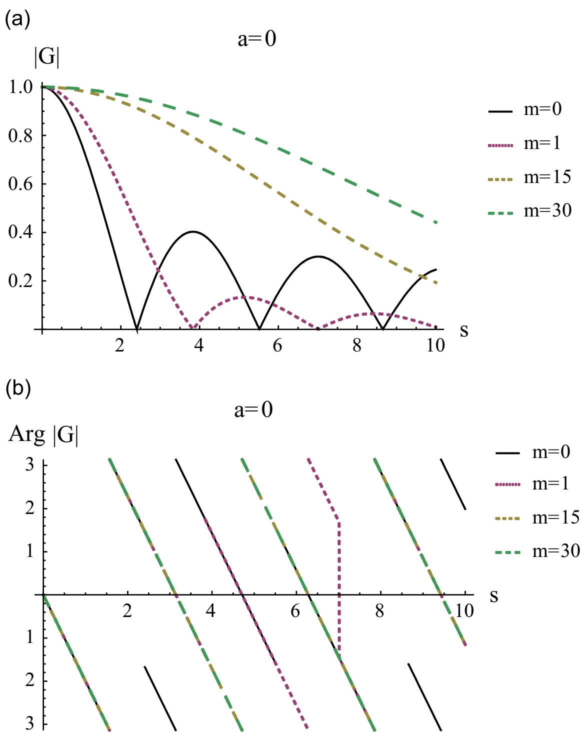

Fig. 1. The waveforms of radial modes are specified by a Bessel function (Eq. (10a)) in the range (Eq. (32b)) in the absence of rotation $a=0$ in Eq. (17a), for the azimuthal orders (Eq. (34a)).