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Thermoelastic damping in microrings with circular cross-section

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ABSTRACT

Predicting thermoelastic damping (TED) is crucial in the design of high *Q* micro-resonators. Microrings are often critical components in many micro-resonators. Some analytical models for TED in microrings have already been developed in the past. However, the previous works are limited to the microrings with rectangular cross-section. The temperature field in the rectangular cross-section is one-dimensional. This paper deals with TED in the microrings with circular cross-section. The temperature field in the circular cross-section is two-dimensional. This paper first presents a 2-D analytical model for TED in the microrings with circular cross-section. Only the two-dimensional heat conduction in the circular cross-section is considered. The heat conduction along the circumferential direction of the microring is neglected in the 2-D model. Then the 2-D model has been extended to cover the circumferential heat conduction, and a 3-D analytical model for TED has been developed. The analytical results from the present 2-D and 3-D models show good agreement with the numerical results of FEM model. The limitations of the present 2-D analytical model are assessed.

1. Introduction

Thermoelastic damping (TED) has been identified as an important mechanism of energy dissipation in vacuum-operated micro-resonators [1,2]. TED is a mechanism of structural damping which originates from the irreversible thermal flux generated due to expansion/compression of elastic structures. The design of micro-electro-mechanical systems (MEMS) requires accurate determination of TED. In 1937 and 1938, Zener [3,4] first developed the theory of thermoelastic damping in a vibrating beam. Zener has obtained two simple but accurate models for TED in the rectangular and circular cross-section beams respectively. The two simple models are [3,4]

$$Q^{-1}_{\text{Zener\_}R} = \Delta_E \frac{\omega\tau_R}{1+(\omega\tau_R)^2} \tag{1}$$

$$Q^{-1}_{\text{Zener\_}C} = \Delta_E \frac{\omega\tau_C}{1+(\omega\tau_C)^2} \tag{2}$$

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where $\Delta_E = \frac{E\alpha^2 \hat{T}_a}{C_v}$, C_v is the specific heat, k_0 is the thermal conductivity, E is the isothermal Young's modulus, α is the thermal expansion coefficient, $\hat{T}_a$ is the absolute ambient temperature, $\tau_R = \frac{h^2 C_v}{\pi^2 k_0}$ and $\tau_C = 0.295\frac{r_0^2 C_v}{k_0}$ are the relaxation times of the two beams, the subscript "R" refers to the rectangular cross-section beam with a thickness of h, the subscript "C" refers to the circular cross-section beam with a radius of r_0. In 2000, Zener's model has been improved by Lifshitz and Roukes [5]. They presented a new function to approximate the temperature field in the rectangular cross-section beam. However, the model developed by Lifshitz and Roukes (LR) is valid only for the rectangular cross-section beam.

Zener's model and LR's model are now widely used to predict TED in microbeam-based resonators. As MEMS technologies evolving, there is an increasing use of microring in micro-resonators. For example, the thin silicon rings are the components of many vibratory micro-gyroscopes [6,7]. The microrings in these gyroscopes are often operated at the in-plane flexural-mode. Some analytical models [8–11] for the TED in microrings with the in-plane vibration have been developed. Next, we summarize the previous works for TED in the microrings.

In 2004, Wong et al. [8] first used Zener's model to predict the TED in microrings undergoing in-plane oscillation. Comparison of the results from Zener's model with experimental data showed that Zener's model can provide realistic predictions of TED in silicon microrings. The relationships between mean radius, radial thickness and TED are explored based on Zener's model.

In 2006, Wong et al. [9] restudied and derived a comprehensive expression for TED in thin ring undergoing in-plane oscillation. Only the heat conduction along the radial thickness of the microring was considered in their model. They found that the two classical models (Zener and LR's models) for microbeam are also reasonable for TED in thin rings. A design method to reduce damping loss in microrings was also developed by Wong et al. [9].

In 2010, Kim et al. [10] investigated TED in a rotating thin ring. The temperature field in the rotating thin ring is obtained. A new model for TED in the rotating ring is developed based on the eigen-value analysis. Their model indicated that increment of the rotating speed of the microring increases the Q-factor. The rotating effect on the TED cannot be neglected for the microrings with a high rotating speed.

The above three papers [8–10] are all based on the approach developed by Zener [3,4]. In Zener's approach, only the heat conduction along the beam thickness was considered. In the above three papers [8–10], only the heat conduction along the radial thickness of the microring was considered. In 2015, we [11] developed a more accurate model for the TED in microrings. The heat conduction along the radial thickness and the heat conduction along the circumferential direction are all considered in our model.

However, all the previous works [8–11] are valid only for the microrings with rectangular cross-section. There are few works on modeling of the TED in the microrings with circular cross-section. This paper develops a new model for thermoelastic damping in the microrings with circular cross-section. The temperature field in this paper is three-dimensional.

It should be emphasized that the normal methods used for micromachining are planar technologies that lead to structures with rectangular cross-sections. For the same reason, it is difficult to make micro-scale structures with circular cross-sections. Therefore, this work is not of immediate relevance to current micro-scale technologies. However, micromachining technology is quickly improving. For example, some micro/nano-beams with elliptical and triangular cross-sections have already been reported in the literatures [12,13]. Therefore this work can be used to predict the TED in the future devices with circular cross-section.

This work is organized as follows. Section 2 first presents a 2-D analytical model for TED in the microrings with circular cross-section. Only the two-dimensional heat conduction in the circular cross-section is considered. The heat conduction along the circumferential direction of the microring is neglected. Then the 2-D model has been extended to cover the circumferential heat conduction, and a 3-D analytical model has been developed. Section 3 provides the simulation results using the present 2-D and 3-D models for TED in microring resonators. The numerical results obtained by the FEM model are also given for comparison. The differences between the present 2-D and 3-D models are discussed. Finally, the principal results of this work are concluded in Section 4.

2. Problem formulation

Consider a microring with circular cross-section, as is shown in Fig. 1. R_0 is the mean radius of the microring and r_0 is the radius of the cross-section. Microrings are capable of both in-plane and out-of-plane flexible vibrations. But microrings are often operated at the in-plane flexural-mode. Therefore only the in-plane vibration is considered in this paper. The microrings in MEMS devices are often thin rings ($R_0 \gg r_0$). In these cases, the Euler–Bernoulli assumptions for the bending beam can be applied to the thin ring vibrating in the in-plane flexural-mode. The main assumptions are as follows. The plane cross-sections in the microring remain plane and perpendicular to the neutral surface during bending. The effects of shear deformation and rotary inertia are not included.

As shown in Fig. 1(a) and (b), a global polar coordinate system (R, θ, Z) is attached to the microring. The origin of the global polar coordinate system is located at the center of the microring. A local coordinate system (x, y, z) is attached to the