Journal of Sound and Vibration 358 (2015) 199–216

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Constitutive sensitivity of the oscillatory behaviour of hyperelastic cylindrical shells

D. Aranda-Iglesias, G. Vadillo, J.A. Rodríguez-Martínez*

Department of Continuum Mechanics and Structural Analysis, University Carlos III of Madrid, Avda. de la Universidad 30, 28911 Leganés, Madrid, Spain

ARTICLE INFO

Article history:
Received 16 January 2015
Received in revised form
30 July 2015
Accepted 31 July 2015
Handling Editor: W. Lacarbonara
Available online 28 August 2015

ABSTRACT

Free and forced nonlinear radial oscillations of a thick-walled cylindrical shell are investigated. The shell material is taken to be incompressible and isotropic within the framework of finite nonlinear elasticity. In comparison with previous seminal works dealing with the dynamic behaviour of hyperelastic cylindrical tubes, in this paper we have developed a broader analysis on the constitutive sensitivity of the oscillatory response of the shell. In this regard, our investigation is inspired by the recent works of Bucchi and Hearn (2013) [28,29], who carried out a constitutive sensitivity analysis of similar problem with hyperelastic cylindrical membranes subjected to static inflation. In the present paper we consider two different Helmholtz free-energy functions to describe the material behaviour: Mooney–Rivlin and Yeoh constitutive models. We carry out a systematic comparison of the results obtained by application of both constitutive models, paying specific attention to the critical initial and loading conditions which preclude the oscillatory response of the cylindrical tube. It has been found that these critical conditions are strongly dependent on the specific constitutive model selected, even though both Helmholtz free-energy functions were calibrated using the same experimental data.

1. Introduction

The analysis of the dynamic behaviour of incompressible hyperelastic shells aroused the interest of the scientific community thanks to the pioneering works of Knowles [1,2]. In these papers Knowles investigated, for the first time, the large-amplitude radial oscillations of a very long, thick-walled cylindrical tube. Namely, the problem of arbitrary amplitude free oscillations was considered in [1] and the problem of forced oscillations with Heaviside step pressure boundary condition was explored in [2]. Shortly after, Zhong-Heng and Soleki [3] inspected the large amplitude vibrations of thick-walled spherical hyperelastic incompressible bodies. The work of Zhong-Heng and Soleki [3] was later revisited, and adapted to the thin-walled spherical shell by Wang [4].

In any of these seminal works, due to the incompressibility of the material, the problem at hand was reduced to that of an autonomous motion of a system with a single degree of freedom. Thus, the emphasis of Knowles [1,2], Zhong-Heng and Soleki [3] and Wang [4] was on obtaining exact expressions for the period of oscillations while, due to the severe (geometrical and material) nonlinearity of these problems [5], the actual states of strain and stress were not determined. Significant efforts were made over the following years to get rid of this incompleteness and provide a full description of the

* Corresponding author. Tel.: +34 916248460; fax: +34 916249430.
E-mail address: jarmarti@ing.uc3m.es (J.A. Rodríguez-Martínez).

http://dx.doi.org/10.1016/j.jsv.2015.07.031
0022-460X/

stress, strain and displacement fields of the problems at hand. In this regard, it is worth mentioning the work of Nowinski and Wang [6] and the series of papers by Shahinpoor and co-workers [5,7–10] that were focused on obtaining complete solutions for the full set of field variables involved in these problems. The great interest raised during the 60s and 70s by the nonlinear oscillations of hyperelastic shells has continued to the present, as detailed in the recent review of Alijani and Amabili [11].

Thus, in recent years, we have to emphasize the contributions of Beatty [12,13], Verron et al. [14,15] and Humphrey and co-workers [16,17] who investigated the radial oscillations of thick- and thin-walled cylindrical and spherical shells subjected to dynamic inflation. The loss of oscillatory behaviour was identified as a key factor which limits the capacity of hyperelastic shells for withstanding large deformations under dynamic loading. Uncovering the causes which lead to the loss of oscillatory behaviour of this type of structures is a crucial issue for different applications. For instance, the dynamic inflation of hyperelastic shells has been lately revisited with the aim of understanding the growth and rupture of arteries and aneurysms [18,19] or to develop electro-mechanical actuators that may be used as reciprocating and peristaltic pumps [20].

All these authors that are cited in previous paragraphs have raised (at least up to some extent) the key role played by the constitutive model in the dynamic response of the hyperelastic shells, and specifically in the critical conditions which lead to the loss of oscillatory behaviour of the structure. In this regard, we have to mention the latest works of Gonçalves et al. [21] and Soares and Gonçalves [22,23] who made a thorough investigation of the nonlinear vibrations of circular, annular and rectangular hyperelastic membranes. The authors showed the constitutive sensitivity of these problems using different strain energy functions calibrated with the same experimental results. In the words of Soares and Gonçalves [23] *the choice of an appropriate constitutive law is a key step in the mathematical modelling of hyperelastic materials*. This statement is further supported by Selvadurai [24] who studied the role played by the constitutive model on the deflection of hyperelastic membranes. Selvadurai [24] concluded that the selection of an appropriate (accurate) strain energy function becomes especially relevant when the objective is to model the large strain behaviour of hyperelastic solids. Moreover, this conclusion is in line with the main outcome derived from the work of Lacarbonara et al. [25] who showed the key role played by the material nonlinearity in the flexural vibrations of elastic rings. In addition, Antman and Lacarbonara [26] and Lacarbonara and Antman [27] have shown the critical influence that material compressibility and viscosity have on the radial motions of cylindrical and spherical shells.

Thus, moved by these works which pointed out the constitutive sensitivity of the oscillatory behaviour of hyperelastic shells, in this investigation we revisit the original problem of Knowles [1,2] and consider free and forced radial oscillations of cylindrical incompressible hyperelastic tubes. Two constitutive models, Mooney–Rivlin and Yeoh, calibrated with the same set of experimental data (see [28,29]) are taken into consideration. A methodical confrontation of the results obtained from both constitutive models raises their influence on the dynamic response of the cylindrical shell. Thus, we have obtained the initial, loading and geometrical conditions which, depending on the constitutive model, impede the oscillatory response of the shell. In addition, for the specific cases in which the shell shows periodic motion we have explored the influence of the constitutive model in the amplitude and period of the oscillations. Further, a complete description of the strain field of the shell is provided for selected loading cases. The salient feature of this paper is to develop an extensive and meticulous investigation to show the constitutive sensitivity of the oscillatory response of hyperelastic cylindrical shells to the full extent.

2. Problem formulation

In this section we derive the differential equations of motion describing the radial oscillations (of large amplitude) of a thick-walled cylindrical shell. The shell material is taken to be incompressible and isotropic within the framework of finite nonlinear elasticity. The main features of the mathematical derivation are presented, while further details can be found in the seminal works of Knowles [1,2].

Let (R_i, R_o) and (r_i, r_o) denote the inner and outer radii of the tube in the undeformed configuration and in the deformed configuration, respectively. Let $\{R, \Theta, Z\}$ denote the coordinates of a point in the tube in the undeformed state with reference to a fixed cylindrical coordinate system coaxial with the cylinder. A particle that was at $\{R, \Theta, Z\}$ in the undeformed state is assumed to have cylindrical coordinates $\{r(R,t), \theta, z\}$ at time t. Plane strain is considered (cylindrical shell of infinite length) and thus $z = Z$. The conservation of linear momentum in the radial direction leads to

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} = \rho \ddot{r} \tag{1}$$

where σ_r and $\sigma_\theta = \sigma$ are the radial and circumferential Cauchy stresses respectively, ρ is the material density and a superposed dot denotes differentiation with respect to time. In this work we identify the following dimensionless variables:

$$\tau = \frac{t}{t_0}, \quad \overline{\sigma}_r = \frac{\sigma_r}{C_{M1}}, \quad \overline{\sigma} = \frac{\sigma}{C_{M1}}, \quad \overline{R} = \frac{R}{R_i}, \quad \overline{r} = \frac{r}{R_i}$$

where $t_0 = R_i \sqrt{\rho / C_{M1}}$, with C_{M1} being a material constant as further discussed in Section 3. Thus, Eq. (1) takes the following non-dimensional form:

$$\frac{\partial \overline{\sigma}_r}{\partial \overline{r}} + \frac{\overline{\sigma}_r - \overline{\sigma}}{\overline{r}} = \ddot{\overline{r}} \tag{2}$$

where now a superposed dot denotes differentiation with respect to the dimensionless variable τ.