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The role of nonlinear torsional contributions on the stability of flexural–torsional oscillations of open-cross section beams

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ABSTRACT

An open-cross section thin-walled beam model, already developed by the authors, has been conveniently simplified while maintaining the capacity of accounting for the significant nonlinear warping effects. For a technical range of geometrical and mechanical characteristics of the beam, the response is characterized by the torsional curvature prevailing over the flexural ones. A Galerkin discretization is performed by using a suitable expansion of displacements based on shape functions. The attention is focused on the dynamic response of the beam to a harmonic force, applied at the free end of the cantilever beam. The excitation is directed along the symmetry axis of the beam section. The stability of the one-component oscillations has been investigated using the analytical model, showing the importance of the internal resonances due to the nonlinear warping coupling terms. Comparison with the results provided by a computational finite element model has been performed. The good agreement among the results of the analytical and the computational models confirms the effectiveness of the simplified model of a non-linear open-cross section thin-walled beam and overall the important role of the warping and of the torsional elongation in the study of the one-component dynamic oscillations and their stability.

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1. Introduction

The nonlinear dynamic behavior of beams is a topic widely discussed in mechanics. While the early works addressed only planar motion, in the seventies the interest was directed to the three-dimensional behavior of beams. In one of the pioneering papers [1], a one-dimensional polar model of a compact beam is used to describe the non-planar, nonlinear dynamics of an inextensional beam. In the model presented, the torsional component was statically condensed, and the warping neglected. Subsequent papers proposed enriched models, some of which included linear warping, as [2]; however, although the linear warping contribution was considered, in most of the cases beams with compact cross-section were analyzed. Among these paper, flexural–torsional–extensional couplings in the motion of a cantilever beam have been considered in [3–5]. In [6], the complexity deriving from tackling nonlinear warping led to the choice to completely neglect it, thus limiting the applicability of the study to some open cross-section beams with comparable bending and torsional

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frequencies. Also the papers based on an extension to the nonlinear Vlasov field theory [7], as [8–10], had to rely on several simplifying assumptions such as considering nonlinear flexural quantities of higher order terms compared to torsional deformational quantities. More recently, in [11], the general formulation for torsional–flexural analysis of beams that make use of two-dimensional St. Venant warping functions and one-dimensional independent warping parameters was solved numerically by the finite element method. Some papers, [12–14], addressed the problem of the general flexural–torsional vibration of homogeneous beams of arbitrarily shaped cross-section under arbitrary loading, numerically solving the boundary value problem with a boundary element method approach (BEM).

In [15] a modified Vlasov theory is proposed to study the dynamic behavior of thin-walled and variable open section beams. The formulation makes use of the state variable approach in the frequency domain and accounts shear strains and rotatory inertias. An experimental analysis for the study of the free vibrations of nonsymmetrical thin walled beams that also provide a benchmark for [15] is proposed in [16]. The same benchmark has been used more recently in [17] to validate results obtained with a one-dimensional model and to investigate the effects of a coarse warping descriptor on the linear dynamics of a clamped–clamped thin-walled beam with non-symmetric cross-sections. Models accounting for non-uniform warping including the effects of torsion and shear forces are formulated in [18,19], for thin-walled and general cross-sections respectively. However, in [19], starting from the assumptions that the cross-section maintains its shape and including three independent warping parameters, distortion was not included even if may be significant for thin-walled cross-sections; some analytical and numerical result are presented in [20]. A linear beam model, where the effects of non-uniform warping related to shear and torsion are considered, is formulated in [21], under the assumption of rigid motion of the section. In [22] the same authors use the Implicit Corotational Method to derive a geometrically nonlinear model for generic cross-sections, which also accounts for non-uniform warping. A finite element is developed to analyze cantilevers with different thin-walled cross-sections. Subsequently a higher order beam model for thin-walled structures that considers higher order warping modes obtained solving the nonlinear eigenvalue problem of the corresponding beam governing equations is presented in [23]

In [24,25] the description of the mechanical behavior of beams with open cross-sections is dealt with using a nonlinear beam model developed rigorously starting from an internally constrained three-dimensional continuum, in which torsional and flexural curvatures of the same order of magnitude are considered. The warping is obtained extending the Vlasov theory [7] to the nonlinear field. The effects of the torsional curvature on the elongation of the longitudinal fibers and the nonlinear warping of the section are considered. Due to the complexity of the model and to the several resonance conditions involved in the mechanical system investigated, it has not yet been possible to appreciate completely the role of the new nonlinear contributions.

Within a class of open cross section beams with certain geometrical and mechanical characteristics, it can be shown that torsional curvature is greater than the flexural ones. A notable simplification is then obtained in the kinematical relations with respect to the model developed in [24], as shown in [26]. With the model developed in [26], it has been possible to understand the role of the new nonlinear contributions in the static behavior of a cantilever beam and in the stability of static solutions. Numerical and experimental investigations have confirmed the results provided by this model.

Although this paper starts with the same assumptions of [26], here the attention is focused on the dynamic behavior of the beam. A simplified set of equations of motion capable of describing the nonlinear dynamics of the open-cross thin-walled beam, also accounting for the nonlinear contribution of the warping, are written for the first time with respect to the cited previous work of the authors and the existing literature. However, it is shown that the reduced equations of motion are capable of preserving the same dynamic characteristics of the rigorous model [24], and consequently are perfectly capable of capturing the main phenomena of the nonlinear response.

Three simplified equations of motion are derived to describe the dynamics of inextensional and shear undeformable nonlinear 3D open-cross section thin-walled beam with a section that presents only one symmetry axis. A Galerkin discretization is performed by using a suitable number of shape functions, chosen with special care. The attention is focused on the dynamic response of the beam to a harmonic force, applied at the free end of the cantilever beam and directed along the symmetry axis of the section, which leads to a one-component oscillation.

The primary objective of this paper is the study of the stability of the one-component oscillation, by investigating the role of the internal resonances related to the nonlinear warping coupling terms. The results obtained by the analytical model are compared with those provided by a computational finite element model. The behaviors ascertained with and without considering warping are compared to highlight the effects of the nonlinear contributions due to the torsional elongation and the warping.

The investigation here performed permits novel stability phenomena to be highlighted, noting that these are strictly connected to the new nonlinear terms due to nonlinear warping and torsional elongation effects.

2. Mechanical model and equations of motion

The analytical model (*AM*), described in [24,25], is here considered to describe the behavior of a thin-walled beam. The beam is assumed to be slender, initially straight, with an open, monosymmetric cross-section and arbitrary restrained at the ends. In order to obtain a model capable of adequately describing the behavior of the beam and, at the same time, of pointing out the main contributions in the equations of motion, the same hypotheses assumed in [26] have been introduced. These hypotheses allow a valid simplification of the displacement field and an easier identification of the leading terms in

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