Journal of Sound and Vibration 340 (2015) 95–111

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Stochastic unilateral free vibration of an in-plane cable network

Gian Felice Giaccu [a], Bernardo Barbiellini [b], Luca Caracoglia [c,*]

[a] Department of Architecture, Design and Urban Planning, University of Sassari, 07041 Alghero, Italy
[b] Department of Physics, Northeastern University, Boston, MA 02115, USA
[c] Department of Civil and Environmental Engineering, 400 Snell Engineering Center, Northeastern University, 360 Huntington Avenue, Northeastern University, Boston, MA 02115, USA

ARTICLE INFO

Article history:
Received 20 July 2014
Received in revised form
25 October 2014
Accepted 1 December 2014
Handling Editor: W. Lacarbonara
Available online 27 December 2014

ABSTRACT

Cross-ties are often used on cable-stayed bridges for mitigating wind-induced stay vibration since they can be easily installed on existing systems. The system obtained by connecting two (or more) stays with a transverse restrainer is designated as an "in-plane cable-network". Failures in the restrainers of an existing network have been observed. In a previous study [1] a model was proposed to explain the failures in the cross-ties as being related to a loss in the initial pre-tensioning force imparted to the connector. This effect leads to the "unilateral" free vibration of the network. Deterministic free vibrations of a three-cable network were investigated by using the "equivalent linearization method".

Since the value of the initial vibration amplitude is often not well known due to the complex aeroelastic vibration regimes, which can be experienced by the stays, the stochastic nature of the problem must be considered. This issue is investigated in the present paper. Free-vibration dynamics of the cable network, driven by an initial stochastic disturbance associated with uncertain vibration amplitudes, is examined. The corresponding random eigen-value problem for the vibration frequencies is solved through an implementation of Stochastic Approximation, (SA) based on the Robbins–Monro Theorem. Monte-Carlo methods are also used for validating the SA results.

1. Introduction

1.1. Context and motivation

Inclined stays on modern cable-stayed bridges are susceptible to wind-induced oscillations (e.g., [2,3]) due to their lengths and slenderness. Several models exist for predicting the loading mechanisms. Typical examples are "dry galloping" (e.g., [4,5]) causing large-amplitude oscillation and "rain-wind induced" oscillation (see, for example, [6] for a recent review). Moreover, other excitation mechanisms can influence the dynamics of the stays, such as various cable-deck interaction phenomena, linear or nonlinear (e.g., [7–13]). Nevertheless, the loading estimation on stay cables still remains an open and partially unresolved issue.

* Corresponding author. Tel.: +1 617 373 5186; fax: +1 617 373 4419.
E-mail addresses: gf.giaccu@uniss.it (G.F. Giaccu), bba@neu.edu (B. Barbiellini), lucac@coe.neu.edu (L. Caracoglia).

http://dx.doi.org/10.1016/j.jsv.2014.12.004
0022-460X/

Nomenclature

Main symbols are only listed. For other symbols, refer to [1]

- a — step size coefficient of the Stochastic Approximation (SA) algorithm
- a_q — gain parameter of the SA algorithm at step q
- $e(\alpha_{E,q})$ — uncertainty term, contaminating $g(\alpha_{E,q})$ at step q of the SA algorithm.
- $g(\alpha_{E,q})$ — noiseless function, associated with the SA algorithm at step q and with $\alpha_{E,q}$
- $k_{MOD,j}(\alpha)$ — modified stiffness of the cross-tie segment, installed between stays j and $j+1$ (Eq. (7))
- $Q_\lambda(\alpha_E)$ — characteristic polynomial function of the equivalent eigen-value/eigen-vector problem, $Q_\lambda(\alpha_E)=\det(\mathbf{Q}(\alpha_E))$, as a function of frequency α_E and depending on parameter λ
- $\widetilde{Q}(\alpha)$ — ensemble average function of the sequence of characteristic polynomial functions (Eq. (14))
- α_E, α — random dimensionless frequency ("E", linearized system)
- $\alpha_{E,q}$, α_q — random dimensionless frequency of the linearized system at step q
- $\overline{\alpha}$, $\overline{\alpha}_\infty$ — true value of the expected value of α
- $\overline{\alpha}_q$ — expected value of the dimensionless frequency at step q (recursive formula)
- ΔK_j — random performance coefficient of the cross-tie segment, installed between stays j and $j+1$
- $\overline{\Delta K_j}$ — mean value of the random ΔK_j
- $\Delta K'_{1,\overline{\alpha}_E}$ — first derivative of the function ΔK_j ($j=1$) with respect to α_E, evaluated at $\alpha_E=\overline{\alpha}_E$
- $\Delta K''_{1,\overline{\alpha}_E}$ — second derivative of the function ΔK_j ($j=1$) with respect to α_E, evaluated at $\alpha_E=\overline{\alpha}_E$
- δ_{SA} — exponent in the formula of the gain parameter a_q
- $\varepsilon_{\Delta K_1}$, $\varepsilon_{\Delta K_1,\mathrm{rel}}$ — absolute (Eq. (A.3)) and relative (Eq. (A.4)) approximation error due to the hypothesis $E[\Delta K_j(\alpha_E)]\cong\Delta K_j(E[\alpha_E])$
- λ — random dimensionless vibration amplitude
- λ_q — dimensionless amplitude, element q of the random sequence
- λ_u — upper limit value of the λ random variable
- $\Upsilon_q(\alpha_{E,q})$ — noisy function of the variable $\alpha_{E,q}$, evaluated by the SA algorithm at step q

Subscripts

- j — generic stay-cable index
- q — index designating the element of a random sequence, also used in recursive SA formulas to designate the iteration step
- m — sample size of the random sequence of λ

Cross-ties are often employed for vibration reduction in the stays since their installation on existing systems is simple. Therefore, accurate prediction of cable network dynamics is of great relevance for the design of mitigation systems on cable-stayed bridges. Even though the emphasis of this study is on cable-cross-tie systems, we provide a short description of other methods for vibration control for the sake of completeness. Mitigation may also be achieved by using, as an alternative, damping devices. The use of damping devices, attached to a stay and anchored to the deck, has been proposed by various investigators (e.g., [14,15]). Recent studies have analyzed the use of more than one damper attached to the stay (e.g., [16]), nonlinear dampers (e.g., [17–19]) and semi-active dampers (e.g., magneto-rheological, [20]).

Finite-element analysis has traditionally been used to study the dynamics of cross-ties [21,22]. An analytical model, based on linear taut-cable theory [23], was proposed to study the in-plane free vibration of cable networks and for predicting the response of real systems (e.g., [24]). This linear analytical formulation has also been considered and used by other investigators to further examine the cross-tie adequacy in controlling wind-related oscillation (e.g., [25]). Since suppression of stay vibration in the plane orthogonal to the plane of the stays is limited (e.g., [24,25]) dynamic models, which include out-of-plane behavior, are not necessary.

To date few studies have addressed the relevant question of cross-tie performance in the presence of nonlinear dynamic connectors, i.e., restrainers with nonlinear restoring effects [26]. More recently, nonlinear dynamic response of a network at incipient failure, due to snapping or slackening of the transverse restrainers, has gained the attention of the researchers. A new model was developed by the authors [27] for the prediction of oscillations on a cable network in the presence of a nonlinear restoring-force effect in the connectors. The "Equivalent Linearization Method" (ELM) was used [27,28]. It was found that, by comparing the ELM solution to direct time-domain integration, the ELM is still accurate for predicting the free vibration. In a previous study [1] the nonlinear effect associated with the incipient slackening in the restrainer due to the loss of the pre-tensioning force, initially imparted to the restrainers, was analyzed in more detail. "Unilateral behavior" was employed to simulate extreme conditions in the restrainer at slackening. This model can reproduce the unilateral restoring-force trend in the cross-ties by using a dimensionless pre-tensioning parameter $\tau_{0,1}>0$, which sets the initial level of pre-stressing force in the connector. The quantity $\tau_{0,1}$ was coined to evaluate the unilateral performance of a spring-type mechanistic model of the cross-tie (e.g., [24]). The model operates by linearization of the system of differential dynamic equations (ELM). The algorithm estimates a "performance coefficient" of the cross-tie (ΔK_1) as a function of vibration amplitude λ of the cable network and of $\tau_{0,1}$. The model was employed [1] to find the minimum level of $\tau_{0,1}$, needed to preserve linearity in the cross-tie response, depending on the vibration amplitude parameter λ of the system. It must be noted that the amplitude parameter λ was coined in [1] to indicate a dimensionless ratio between maximum modal amplitudes in the stays during unilateral free vibration, induced by nonlinear behavior in the cross-tie. This quantity must not be confused with