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The use of modal curvatures for damage localization in beam-type structures

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ABSTRACT

The localization of stiffness variation in damaged beams through modal curvatures, i.e., second derivative of mode shapes, is studied by exploiting a perturbative solution of the Euler–Bernoulli equation. It is shown that for low order modes the difference between undamaged and damaged modal curvatures has only one distinct peak if the damage is localized in a narrow region. This phenomenon is independent of the presence of experimental noise and of the technique used to reconstruct the curvature mode shapes from the displacement mode shapes. Broader damages cause the modal curvature difference to have several peaks outside the damage region that could result in a false damage localization. The same effect is present at higher modes for both narrow and broad damages. As a result, modal curvatures can be effectively used to localize structural damages only once they have been properly filtered. Here the perturbative solution is used to introduce an effective damage measure able to localize correctly narrow and broad damages and also single and multiple damages cases.

1. Introduction

In damage detection techniques different quantities can be selected to be observed. The majority of approaches use the variations of natural frequencies: they are easily measurable with high accuracy, but as global quantities are less sensitive to concentrated damages [1–5]. In comparison, the advantage of using mode shapes and their derivatives as a basic feature for damage detection is a well-established result. First, mode shapes contain local information, which makes them more sensitive to local damages and enables them to be used to detect multiple damages or when nonlinearities are present in the structural response [6–9]. Second, the mode shapes are less sensitive to environmental effects, such as temperature, than natural frequencies [10–12].

Over the years several damage identification techniques have exploited the use of changes in Modal Displacements (MDs) between the undamaged and the faulty system. Such techniques have been applied with different degrees of success to structures such as rods [13], beams [14,15], plates and even more complex systems (see, for instance, [16–18]). Their major limitation is that, even if the damage is localized, its effect on MDs can spread all over the structure. As a consequence, it can be cumbersome to identify the actual damage position when several peaks in the MD difference appear.

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On the contrary, Modal Curvatures (MCs) are widely considered a superior indicator of damage position, as for localized damages, MC difference is thought to be concentrated in the damaged area [19–25]. In [10] it is shown that the MC shape-based methods are much more robust and promising than the MD shape-based methods, mainly when curvatures are directly measured from the strain mode shape. However, some authors [20,26] pointed out that MCs fail to provide a clear localization of the damage position even in simple cases such as cantilevered beams. Montalvao [26] for instance state that "higher derivatives are more promising for damage identification, but […] false damage indications may be observed at mode shape nodal points or where the quality of the measurements is relatively poor".

One of the first attempts to identify damages through MC changes was presented in [19]. Through numerical examples on a simply supported beam, they show that MCs are mainly localized in the region of damage compared to the changes in the MDs. However, major oscillations appeared when they compared changes in MCs for higher modes. They also stated that a relatively dense measurement grid is required in order to get a good estimate of the curvatures for higher order modes.

Differences in MCs have been applied in [20] to detect damage in bridges. The authors comment on the numerical simulations in [19]: "the difference in MC between the intact and the damaged beam showed not only a high peak at the fault position but also some small peaks at different undamaged locations for the higher modes. This can cause confusion to the analyst in a practical application in which one does not known in advance the location of faults". They surmised that the incorrect damage position was due to the numerical scheme used in Pandey et al. [19] to calculate MCs from MDs. In trying to overcome this problem, the authors computed modal curvatures by two different algorithms: from the MDs, by using a central difference approximation, and from the bending moment obtained through finite element. As a result, they found that in any case the difference in MCs shows several peaks not only at the position of the damaged element but also at other positions regardless which algorithm was used to evaluate the MCs. In this respect, more advanced filtering techniques such as spline interpolation [18], modified Laplace operator [27] and wavelet transform can be applied to obtain a robust estimate of the MCs from the noise corrupted MDs; however, as shown in the next sections, the presence of peaks in the MC difference outside the damage region is independent of the noise presence yet could affect noise free data.

Lestari et al. [24] computed the eigenvectors and eigenvalues of a damaged beam through a perturbation method. Their solution was later corrected and improved [28]. In all these papers first-order eigenmodes contain a term depending on the undamaged mode shape; indeed, it is shown later in Section 2 that this term has to be filtered out when considering differences between normalized MCs or MDs. As a consequence, their filtering procedure does not provide a clear damage localization as the resulting MC variations have relevant peaks not only in the damaged area but all over the structure. This result is apparently in contrast with the numerical simulations carried out by several authors [19,20,23,25] where, at least for low order modes, the main peak on the MC difference occurs in the damaged region.

A special mention deserves non-baseline techniques based on MCs which have shown promising results [see for instance [29]]; however such techniques normally require large number of measurement points which makes their application to large structures difficult.

In this paper, by using a perturbative solution, it is demonstrated that MCs do not convey localized information on the damage position, if not properly processed. As a consequence, a novel filtering procedure for MCs is introduced that in turn leads to an effective damage localization. By considering numerical examples we show that, for high order modes, the MC difference is unsuitable for damage localization. We remark that this issue is not due to the algorithm used to computed the MCs, as surmised by some authors, but it is intrinsic of the normalized MC difference. The proposed filtering procedure is used to obtain from the MC difference an accurate estimate of single and multiple damage shapes by using only the minimum number of information conveyed in the first few eigenmodes.

2. Modal analysis of damaged beam

In this section we derive a perturbative solution of the Euler–Bernoulli beam equation with non-constant stiffness; such a solution is then used to obtain a closed form expression of the MC difference and introduce a novel filtering procedure for damage localization.

Within the framework of the Euler–Bernoulli beam theory, we assume the damage to be a localized stiffness variation expressed as

$$EI(x) = EI_0(1-\varepsilon\,\eta(x)) \tag{1}$$

where E is Young's modulus of the material and I_0 is the second moment of area of the undamaged beam; $\eta(x)$ is the damage shape, i.e., a smooth continuously differentiable function that is non-zero only in the damaged region, $0 \le |\eta(x)| \le 1$, and ε is the intensity of damage, $0 \le |\varepsilon| < 1$.

The governing equation for the ith transverse mode of displacement $v^*(x)$ of the damaged beam is

$$\frac{d^4 v_i^*(x)}{dx^4} - \varepsilon\,\frac{d^2}{dx^2}\left[\eta(x)\frac{d^2 v_i^*(x)}{dx^2}\right] - \lambda_i^* v_i^*(x) = 0 \tag{2}$$

where v_i^* and λ_i^* are the ith eigenfunction and eigenvalue of the damaged system, respectively. For small damage intensities, they can be expanded as a power series of ε, i.e.

$$v_i^*(x) = v_i^0(x) - \varepsilon\, v_i^1(x) + o(\varepsilon^2), \quad \lambda_i^* = \lambda_i^0 - \varepsilon\,\lambda_i^1 + o(\varepsilon^2) \tag{3}$$