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Semi-analytical solution of random response for nonlinear vibration energy harvesters

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ABSTRACT

Due to the prominent broadband performance of nonlinear vibration energy harvester, theoretical evaluations for the mean-square response to random excitations and the associated mean output power are of great interest. By employing the generalized harmonic transformation and equivalent nonlinearization technique, established here is a semi-analytical solution of random response for nonlinear vibration energy harvesters subjected to Gaussian white noise excitation. The semi-analytical solution for stationary probability density of the system response is obtained by two iterative processes. Numerical results for a Duffing-type harvester demonstrate rapid convergence of the iterative processes and high evaluation accuracy for the mean-square response and the mean output power. Furthermore, the influence of harvesting circuit on the mechanical subsystem can be converted to modified quasi-linear damping and stiffness with energy-dependent coefficients, which is different from the traditional viewpoint on the equivalence of constant-coefficient damping and provides more comprehensive explanation on the influence of harvesting circuit.

1. Introduction

Researches on vibration energy harvesting have been explosively growing due to the miniaturization of electronic devices and ubiquitous existence of ambient kinetic energy [1–4]. Linear vibration energy harvesters, as the first adopted design, depend on the resonant phenomena and have narrow effective frequency bandwidth, operating efficiently only when the excitation frequency matches their fundamental natural frequency. Small perturbation of the excitation frequency from linear harvesters' fundamental frequency drops the energy output dramatically. Thus, linear harvesters based on the principle of linear resonance are not suitable to harvest energy from broadband and random excitation sources which are typical excitation types in realistic environments. Although active and passive tuning mechanisms and harvesting array have been developed to broaden the usable bandwidth of linear harvesters, the former usually requires external power or complex design and the latter reduces power density and adversely affects the scalability [5,6]. Therefore, more advanced solutions are desired to enhance the broadband performance of energy harvesters.

The intentional introduction of stiffness nonlinearity through magnetic attraction/ repulsion and mechanical techniques is a promising approach to constitute nonlinear vibration energy harvesters with broadened frequency bandwidth [5,7–10].

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The stiffness nonlinearity can extend the coupling between harvesters and excitations to a wider range of frequency. The nonlinear harvesters are classified into mono-stable and bistable types. The former depends on the nonlinear resonant behaviors while the latter depends on the inter-well large-amplitude movement. To evaluate the broadband performance of nonlinear harvesters, it is of great interest to establish an effective analytical technique for the random response and mean output power.

A nonlinear vibration energy harvester usually consists of a nonlinear mechanical subsystem coupled to a harvesting circuit through an electromechanical coupling mechanism. For the continuous mechanical subsystems, such as the buckled beams induced by axial force or magnetic attraction [11], the curved plates [12] and the curved shells [13], the governing partial differential equation can be simplified to a second-order ordinary differential equation with nonlinear stiffness terms through the one-mode Galerkin approximation [9,14]. For the lumped-parameter mechanical subsystems, the governing equation is of the form of second-order nonlinear ordinary differential equation. The harvesting circuit is described by a first-order linear ordinary differential equation with respect to electric voltage for piezoelectric mechanism or current for electromagnetic mechanism [9,10,15]. Thus, the research on the random aspect of nonlinear harvesters required quantitative evaluation of the random response for the coupled equations, which compose a second-order nonlinear differential equation and a first-order linear differential equation [10, 16]. The nonlinear property of the mechanical equation itself and the coupled behavior between the mechanical and electric equations, nevertheless, make the analytical prediction a challenging task.

To simplify the analysis of the coupled equations, one may reasonably deal with the nonlinearity or simplify the coupled equations into a reduced second-order nonlinear differential equation. The former is almost impossible as the inappropriate disposition on strong nonlinearity often leads to thorough inaccuracy [17]. The latter is relatively easy to be implemented through some mandatory assumptions. Assume that the effective inductance of the coil in electromagnetic harvesters or the effective capacitance of the piezoelectric element in piezoelectric harvesters can be neglected, the current or voltage can be directly expressed as the function of system velocity through the electric equation and therefore, the coupled equations can be reduced to a second-order differential equation with an effective damping term [18–21]. Bases on the reduced equation, the random response and output power have been investigated analytically through the Fokker–Plank–Kolmogorov (FPK) equation method, with some interesting conclusions [18,19,22]. Also, the equivalent linearization technique has been adopted to provide an explicit expression for the displacement variance which is easier to interpret despite of less accuracy [20,21].

The above procedure to analyze the coupled equations through a reduced equation with an effective damping term is certainly simple. Some of the main conclusions, such as the independence of output power on stiffness nonlinearity, however, are not consistent with the associated realistic system [16]. Solving the coupled equations directly is necessitated to capture the accurate performance of the nonlinear harvesters, and many techniques have been developed regarding this issue. Following the conversion to state-space, some literature studies resorted to numerical integration to evaluate the random response [7,11,23–28]. The integration approach is straightforward and versatile to system and excitation properties. Nevertheless, the whole process of computation is generally time-consuming. To directly derive the response statistics, the moment differential equations have been established from the FPK equation and then closed through the fourth-order cumulant-neglect closure scheme [16]. This method provides a better approximation for the random response of the couple equations, its manipulation, however, is complex due to the multi-solution problem.

The joint probability density function of the system response is governed by the associated multidimensional FPK equation, which represents a linear partial differential equation with varying coefficients. The exact solution of the FPK equation is not easily attainable even for the stationary sense, and some approximate techniques, such as finite element method and Galerkin procedure, have been adopted. Finite element method provides accurate predictions for the random response and mean output power regardless of the shape of the potential energy function, and optimized potential shape has been comprehensively investigated [29,30]. By expanding the stationary joint probability density function into orthogonal polynomials specially adapted to the problem, and minimizing the residual through a Galerkin procedure, the approximate probability density function was derived [31–33]. Numerical results showed that the optimal potential shape exhibits double wells, and both the potential energy barrier and the separation distance between the stable points of the optimal potential shape increase with the excitation intensity. In addition, the equivalent linearization technique was adopted to establish the equivalent linear system of the nonlinear energy harvester, resulting in close-form approximate expression for the mean output power [34,35]. Readers interested in the analytical procedures on the coupled equations and the conclusions on the random response of nonlinear harvesters can refer to the review literature by Daqaq et al. and the references therein [10]. The equivalent linearization technique is the simplest method compared to the numerical integration, moment method, finite element method and Galerkin procedure, however, its Gaussian random process assumption on system states induces essential inaccuracy in evaluating the random response for nonlinear harvesters with strong nonlinearity [36]. Thus, it is necessary to develop a more accurate and succinct technique which keeps the same simplicity of the equivalent linearization technique while avoiding the drawback of Gaussian process assumption.

In this paper, the random response of the nonlinear vibration energy harvester subjected to Gaussian white noise excitation is investigated through the generalized harmonic transformation and equivalent nonlinearization technique. Firstly, the electric quantity is explicitly expressed as the integration of system velocity by solving the electric equation, which is then approximated by the algebraic relation of system mechanical states and system mechanical energy using the generalized harmonic transformation. The above disposition leads to a modified second-order differential equation with