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Direction finding using a biaxial particle-velocity sensor ☆

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ABSTRACT

A particle-velocity sensor measures the acoustic particle velocity, instead of the acoustic pressure. A *bi*axial velocity-sensor (a.k.a. a "u–u probe") consists of *two uni*axial velocity-sensors, oriented orthogonally and (possibly) displaced in space. This paper investigates what orientations of the two axes would allow azimuth-elevation direction finding, what closed-form formulas to achieve this direction finding via eigen-based parameter-estimation algorithms, and what the corresponding Cramér–Rao bounds would be.

1. The acoustic particle velocity field (APVF)

A microphone or a hydrophone, each being an isotropic sensor of acoustic pressure, samples an incident acoustic wavefield as a *scalar* field. Thereby neglected is the underlying vector field of the acoustic particle velocity, which represents a spatial gradient of the pressure scalar field.

Consider a particle-velocity vector field, excited by a unit-power point-size acoustic emitter (in the far field or the near field), impinging from an elevation angle-of-arrival of $\theta \in [0, \pi]$ measured from the positive z-axis, and an azimuth angle-of-arrival of $\phi \in [0, 2\pi)$ measured from the positive x-axis. The resulting unit-power particle-velocity vector equals [2,16]

$$\mathbf{a}^{(\mathrm{PV})} \stackrel{\mathrm{def}}{=} \begin{bmatrix} u(\theta,\phi) \\ v(\theta,\phi) \\ w(\theta) \end{bmatrix} \stackrel{\mathrm{def}}{=} \begin{bmatrix} \sin(\theta)\cos(\phi) \\ \sin(\theta)\sin(\phi) \\ \cos(\theta) \end{bmatrix}. \tag{1}$$

For this acoustic *particle-velocity* vector of (1), each of its Cartesian component represents a first-order spatial derivative (along that Cartesian coordinate) of the pressure field, and may be measured by a *uni*axial particle-velocity sensor aligned in parallel to that Cartesian coordinate. Such particle-velocity sensor technology has been used in underwater acoustics and air acoustics [1] for over a century, and has attracted much renewed interest [3,13]. For comprehensive surveys of the open literature on signal-processing algorithms related to the particle-velocity sensor, refer to [15,18,19].

This paper will focus on a *bi*axial velocity-sensor (a.k.a. a "u–u probe") that comprises *two* identical *uni*axial particle-velocity sensors.[1] The two *uni*axial particle-velocity sensors' possible orientations, if limited to the three Cartesian axes, can

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[1] There also exists the "p–u probe", which is equivalent to one *uni*axial velocity sensor plus one pressure sensor. That "p–u probe", if subject to the two constraints mentioned below, would also have nine different configurations. See [22] for details.

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have $\binom{3}{2}=3$ different configurations (namely x- and y-axes, x- and z-axes, y- and z-axes). Moreover, if these two uniaxial particle-velocity sensors are spatially displaced (as implemented in hardware in [6,9]) along any Cartesian axis (as), that displacement may have 3 possible orientations. Altogether, there are thus $3\times 3=9$ possible configurations. Some such *bi*axial particle-velocity sensors have already been implemented [6,9,10]. Their beam patterns and directivity have been investigated in [14,17]. Their direction-of-arrival estimation capability, however, has not yet been systematically investigated in the open literature.[2] The present work will fill this literature gap, by presenting closed-form formulas for the estimation of an incident source's azimuth-elevation direction-of-arrival. The corresponding Cramér–Rao lower bounds will also be derived here in closed forms. The effect of the biaxial particle-velocity sensor's spatial aperture on its direction finding capability will be analyzed as well.

2. The *bi*axial particle-velocity sensor's measurement model

If the u–u probe's two constituent *uni*axial sensors have orientations limited to only the three Cartesian axes (and are collocated), the resulting *bi*axial particle-velocity sensor's array manifold equals

$$\mathbf{a}^{(\text{collocate})}=\mathbf{S}\mathbf{a}^{(\text{PV})},$$

where $\mathbf{S}$ denotes a 2×3 selection matrix (which has a "1" on each row, but zeroes elsewhere). For example, if the *bi*axial particle-velocity sensor consists of an x-axis oriented *uni*axial particle-velocity sensor and a y-axis oriented *uni*axial particle-velocity sensor, its $\mathbf{S}$ would be

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

Suppose these two constituent *uni*axial sensors are displaced in space, such that one *uni*axial sensor lies at the Cartesian origin and the other *uni*axial sensor lies at $(\delta_x,\delta_y,\delta_z)$, without loss of generality. Then, the *bi*axial particle-velocity sensor's far-field array manifold becomes

$$\mathbf{a}^{(\epsilon-\text{axis})}_{\zeta_1,\zeta_2}=\underbrace{\begin{bmatrix} 1 & 0 \\ 0 & e^{j(2\pi/\lambda)\left(\delta_x\sin(\theta)\sin(\phi)+\delta_y\sin(\theta)\cos(\phi)+\delta_z\cos(\theta)\right)} \end{bmatrix}}_{\stackrel{\text{def}}{=}\mathbf{D}}\mathbf{S}\mathbf{a}^{(\text{collocate})},$$

where the superscript ϵ specifies the displacement-axis between the two uniaxial sensors, $\zeta_i\in\{x,y,z\}$ denotes the ith uniaxial sensor's own orientation, with $i=1,2$. The spatial phase factor of $e^{j(2\pi/\lambda)\left(\delta_x\sin(\theta)\sin(\phi)+\delta_y\sin(\theta)\cos(\phi)+\delta_z\cos(\theta)\right)}$ arises on account of the spatial displacement between the two *uni*axial particle-velocity sensors.

Suppose further that $(\delta_x,\delta_y,\delta_z)\in\{(\Delta_x,0,0),(0,\Delta_y,0),(0,0,\Delta_z)\}$. These 3 possibilities, along with the two *uni*axial sensors' three sets of possible orientations, would make a total of 9 possible configurations for the *bi*axial particle-velocity sensor. For example, an x-axis oriented uniaxial sensor paired with a z-axis oriented sensor (i.e. $(\zeta_1,\zeta_2)=(x,z)$) would have only the three configurations as shown in Fig. 1a–c, with the corresponding array manifold being, respectively,

$$\mathbf{a}^{(x-\text{axis})}_{x,z}=\begin{bmatrix}\sin(\theta)\cos(\phi)e^{j(2\pi\Delta_x/\lambda)\sin(\theta)\cos(\phi)} \\ \cos(\theta)\end{bmatrix},$$

$$\mathbf{a}^{(y-\text{axis})}_{x,z}=\begin{bmatrix}\sin(\theta)\cos(\phi)e^{j(2\pi\Delta_y/\lambda)\sin(\theta)\cos(\phi)} \\ \cos(\theta)\end{bmatrix},$$

$$\mathbf{a}^{(z-\text{axis})}_{x,z}=\begin{bmatrix}\sin(\theta)\cos(\phi)e^{j(2\pi\Delta_z/\lambda)\cos(\theta)} \\ \cos(\theta)\end{bmatrix}.$$

A number of relationships exist among the array manifolds of the nine configurations:

$$\mathbf{a}^{(x-\text{axis})}_{z,x}(\theta,\phi)=\mathbf{a}^{(y-\text{axis})}_{z,y}\left(\theta,\frac{\pi}{2}-\phi\right), \tag{2}$$

$$\mathbf{a}^{(y-\text{axis})}_{z,x}(\theta,\phi)=\mathbf{a}^{(x-\text{axis})}_{z,y}\left(\theta,\frac{\pi}{2}-\phi\right), \tag{3}$$

$$\mathbf{a}^{(z-\text{axis})}_{z,x}(\theta,\phi)=\mathbf{a}^{(z-\text{axis})}_{z,y}\left(\theta,\frac{\pi}{2}-\phi\right). \tag{4}$$

[2] Direction finding using a *bi*axial particle-velocity sensor has been discussed in [11,20], but only for *one*-dimensional (i.e. azimuth-only) direction finding, and only if the two constituent *uni*axial particle-velocity sensors are in spatial collocation. This present paper instead considers the categorically more general scenario of azimuth-elevation *two*-dimensional direction finding, with the two *uni*axial particle-velocity sensors being possibly separated in space.