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Elastic metamaterial-based impedance-varying phononic bandgap structures for bandpass filters

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ABSTRACT

In this study, we propose a novel impedance-variation scheme in a half-quarter-wave stack (HQWS) structure to achieve desired bandpass filtering performances such as quasi-flat tops, steep bandedges and wide surrounding bandgaps. Specifically, only the characteristic impedances of constituent half-wave layers are varied without altering the phase shift of π at the center frequency of the target passband that is the Fabry–Perot resonance frequency of the unperturbed original HQWS structure. Because the simultaneous control of characteristic impedance and phase shift in each of the constituent half-wave layers is not achievable only by layer-sizing, the varied layers must be realized by metamaterials. So, specially-configured aluminum-based metamaterials having a double-slit void inhomogeneity are engineered and the actual wave transmission performance of the metamaterial-realized HQWS structure is examined numerically and experimentally.

1. Introduction

Over the past few decades, photonic/phononic crystal (PC) structures have received a great attention because of their extraordinary characteristics in wave propagation and energy transmission [1–6]. Among these, the Bragg bandgap phenomenon [1,4,7,8] is well-known, which originates from the destructive interference of scattered waves generated at the periodic lattices when wavelengths are in the order of a scale of periodicity. An extensive work has been done on the Bragg bandgap phenomenon and its applications such as filters [9–15]. In this paper, the PC structures utilizing the bandgap phenomenon will be referred to as photonic/phononic bandgap (PBG) structures as in earlier works.

While PBG structures can be used as bandstop reflection filters as well as bandpass filters because of their intrinsic formations of distinctive passband and stopband regions, finite-sized PBG structures cannot exhibit ideal bandpass filter performances due to resonance-induced fluctuations in their transmission spectra [1,4,8]. In order to realize transmission passbands with negligible fluctuations, methods utilizing resonances such as the Fabry–Perot resonance (FPR) in a one-dimensional (1D) PBG structure have been suggested. One can construct a 1D PBG structure by inserting an element of a single quarter-wave phase-shift in the center of the structure [8,16–18] or form a 1D PBG structure consisting of alternating negative-index metamaterial and dielectric slabs [19,20]. The PBG structures with a single quarter-wave phase-shift can

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develop a very narrow transmission peak in the middle of a bandgap region owing to the quarter-wave phase-shift, yielding the FPR. The PBG structures employing negative-index metamaterial slabs can also develop multiple sharp transmission peaks at the FPRs within wide bandgap regions. Although such PBG structures achieve non-fluctuating transmission passbands surrounded by stopbands, they cannot be used when wider passbands with flat transmission tops and steep bandedges are demanded.

To overcome the limitations of the above-mentioned methods, a 1D PBG structure having multiple, symmetrically-distributed quarter-wave phase-shift slabs were proposed [11,21,22]. If the locations of multiple phase-shift slabs inside a PBG structure are optimized, the structure can yield a wide passband with the quasi-flat top. But the size of a PBG structure should be sufficiently large to get a flattened wide passband and the steep bandedges simultaneously, and the bandwidths of surrounding stopbands become rather narrow relative to the bandwidth of the passband.

Motivated by the above-mentioned observations, we aim to engineer 1D PBG structures for realizing quasi-ideal bandpass characteristics: flattened passband, steep bandedges, wide and deep surrounding stopbands. To achieve this objective, we use the well-known half-quarter-wave stack (HQWS) structure [8] with the following propositions: (1) while keeping the material characteristics of quarter-wave layers unchanged, only the characteristic impedances of half-wave layers are varied, and (2) the impedance-varying half-wave layers are realized by the proposed positive-index elastic metamaterials. We choose the HQWS structure because it intrinsically exhibits a bandpass-filter-like transmission spectrum because the half-wave layers contribute to the formation of the FPRs and the coexistence of the half- and quarter-wave layers contributes to the formation of two Bragg bandgaps symmetrically located in both sides of the FPRs. However, the passbands formed around the FPRs are known to suffer from fluctuations when the HQWS structure is finite. Therefore, it cannot be directly used as an ideal bandpass filter. As a means to suppress the undesirable fluctuations in the passbands and achieve a flattened transmission passband, we propose to vary characteristic impedances of half-wave layers while keeping phase-shifts in the layers unchanged. Note that the proposed impedance-varying and phase-shift maintaining scheme is totally different from conventional geometric (thickness) variations [23–29] (as shall be explained in Section 2). The difficulty in the impedance-only-variation lies in actual realization. So, we will realize the proposed scheme by using positive-index elastic metamaterials. The specific configurations of the metamaterials will be developed through this study and the bandpass filtering performance of a metamaterial-based impedance-varying HQWS structure will be validated through experiments.

The organization of this paper is as follows. The effects of impedance variation on the transmission characteristics of finite PBG structures are presented in Section 2. Section 3.1 shows specific impedance distributions in order to achieve quasi-flat-top passbands and steep bandedges. With applications to bandpass filters in mind, the metamaterial-based realization of the impedance-varying HQWS structures is presented in Section 3.2 and the bandpass filtering performance is validated by experiments in Section 3.3. The findings from this study are summarized in Section 4. In Appendix A, effective properties of the proposed elastic metamaterials are derived.

2. Effects of impedance variation

To begin with, it is emphasized that the proposed approach that varies impedance and yet keeps phase shift unchanged in the constituent layers of a PBG structure is conceptually different from conventional variation approaches perturbing layer thicknesses. To explain this clearly, let us begin with the transfer matrix **T** for 1D longitudinal wave propagation in periodically-layered elastic cells [30]. Specifically, **T** relates velocity (v) and normal stress (σ) at the right side of the $(m-1)$th cell and those at the right side of the mth cell:

$$\begin{Bmatrix} v \\ \sigma \end{Bmatrix}_m = \mathbf{T}\begin{Bmatrix} v \\ \sigma \end{Bmatrix}_{m-1}. \tag{1}$$

If each unit cell is assumed to consist of two layers made of materials 1 and 2, **T** can be written as a product of two similar matrices:

$$\mathbf{T} = \begin{bmatrix} \cos\varphi_2 & -(\mathrm{i}/z_2)\sin\varphi_2 \\ -\mathrm{i}z_2\sin\varphi_2 & \cos\varphi_2 \end{bmatrix}\begin{bmatrix} \cos\varphi_1 & -(\mathrm{i}/z_1)\sin\varphi_1 \\ -\mathrm{i}z_1\sin\varphi_1 & \cos\varphi_1 \end{bmatrix} \quad \left(\mathrm{i} = \sqrt{-1}\right). \tag{2}$$

In Eq. (2), z_i represents the characteristic impedance of material i and φ_i, the phase shift in the layer made of material i both for longitudinal wave propagation. The phase shift φ_i is related to wavenumber k and layer thickness d as $\varphi_i = k_i d_i$ $(i = 1,\ 2)$. As Eqs. (1) and (2) show, the transfer matrix of a PBG structure can be characterized by two parameters, φ and z only. While the conventional approach of geometric variations is to *vary* d_i, i.e., φ_i *without changing* z_i, the proposed approach is to *vary* z_i *without changing* φ_i.

Let us demonstrate the effects of impedance variation on transmission characteristics by employing the simplest PBG structure, a quarter-wave stack (QWS) structure, commonly used as a Bragg reflector [4,11,16]. As illustrated in Fig. 1(a), a 2-period bi-quarter-wave layered structure is inserted in an infinite medium of impedance z_b. The impedances of low-impedance layers are indicated by $z_a^{(m)}$ (unit cell number $m = 1,\ 2$) while those of high-impedance layers, by z_b with $z_a^{(m)} < z_b$. The phase shifts in low- and high-impedance layers are denoted by $\varphi_a^{(m)}$ and φ_b, respectively with $\varphi_a^{(m)} = k_a^{(m)} d_a^{(m)} = \varphi_b = k_b d_b = \pi/2$ specifically at $\hat{f} = 1$, where $\hat{f}$ is the frequency normalized by the center frequency of the first Bragg bandgap. For subsequent