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Stick-mode duration of a dry-friction oscillator with an uncertain model

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ABSTRACT

This paper investigates the dynamics of a simple dry-friction oscillator which is composed of a block, modeled as a particle, connected to a fixed support by a spring. The block moves over a continuous belt that is driven by rollers. The frictional force between the block and the belt is modeled as a Coulomb friction. Due to this friction model, the resulting motion of the block can be characterized into two qualitatively different modes, the stick- and slip-modes, with a non-smooth transition between them. The focus of the paper is to quantify the percent of time in which the block stays in the stick-mode for different models of periodic belt velocity and for different values of the friction coefficient. Continuous, discontinuous, and random models of belt velocity were considered. The objective is to compare their influence in the duration of the stick-mode. The time of stick represents the fraction of time that the oscillator sticks, it can be viewed as the probability of a biased-coin problem: stick is head and slip is tail.

1. Introduction

The motivation for exploring the different aspects of friction must have had its origin in solving practical problems. Friction is not a new subject. It had been a topic of technological attention long before the dawn of science, and it is still an important topic in science and engineering today [1–5]. Study of oscillatory conditions in dry friction systems has practical significance for providing protection against vibrations, seismic isolation, reduction of noise, and friction-induced vibration, etc. [6,7]. The nature of dynamic friction forces developed between bodies in contact is extremely complex and affected by many factors: the constitution of the interface, inertia and thermal effects, roughness of the contacting surfaces, history of loadings, the presence of lubricants, and many others [8–11]. Besides this, the stick–slip vibrations are self-sustained oscillations induced by dry friction [12] and since the friction can be characterized into two qualitatively different parts (kinetic and static frictions) with a non-smooth transition, the resulting motion also has a non-smooth behavior [13–17]. Two qualitatively different modes, the stick- and slip-modes, characterize the response of a dry-friction oscillator. We call stick when the relative velocity between the bodies in contact is null in a time-interval and we call slip if the relative velocity is non-zero, or zero in isolated points. As these two modes have a non-smooth transition between them, stick–slip systems belong to the class of non-smooth systems, such as systems with stops, impacts, and hysteresis [18–22]. The interest of analyzing the stick–slip dynamics is reflected by the increasing amount of research in this area (see for instance [23–26,20,27–29]). The friction non-smooth behavior associated with the absence of a universally accepted friction model

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given to the friction coefficient is a random behavior [30]. The variability of properties of contacting surfaces due to the influence of ambient conditions (such as temperature, humidity, lubrication, state of the surfaces) makes the stochastic approach the ideal way to deal with dry-friction oscillators.

Stick–slip dynamics appears also in several other situations, for example in the peeling of an adhesive tape from a roller [31], but due to different reasons. Others examples in which stick–slip dynamics appears can be found in [32–34]. We are only interested in stick–slip caused by dry-friction.

The main motivation of this paper is to better understand from a deterministic and from a stochastic viewpoint what are the parameters that control the duration of the stick-mode in a very simple dry-friction oscillator. We believe that the theoretical knowledge on the role of these parameters can provide improvements in the design of mechanical systems and can help in the development of control techniques to reduce the stick duration. These techniques could be implemented in many mechanical systems in which undesirable stick appears. One example of application is on drilling, in which the stick reduces the penetration rate and, following [35], appears during 40 percent of drilling time. Despite the economic interest and multiple engineering applications, few references dealing with the duration of the stick-mode were found after an extensive literature review, as [25]. The system analyzed here is a very simple dry-friction oscillator. It has a block, modeled as a particle, with mass but no dimension; a spring to give an elastic force; and a belt to drive the system. Between the block and the belt there is dry friction. This system is a bare minimum to study stick–slip, but it somehow mimics the dynamics found in the drilling process. On drilling, one of the consequences of the stick-mode is that, since a constant velocity is imposed at the top of the drillstring and during the stick the bit at the bottom does not move, the drillstring is twisted and acts as a spring. Thus, there is an equivalence between parameters of system analyzed in this paper and the parameters found in the drilling process. The spring acts like the drillstring, the mass like the drill bit mass and the belt velocity like the imposed velocity at the top of the drillstring. Besides this, both systems are subjected to the same phenomenon: the stick- and slip-modes. These equivalences are some of the motivations of the paper. We do believe that the system been studied in this paper is, at least qualitatively, similar to the stick–slip found in the drilling process. Of course, in drilling the problem is harder due to the complexity of the geometry and side conditions. In the example we study, we look at the bare minimum.

The equation of motion of the system has different roles during stick and slip. Indeed, in the stick-mode, the block moves with the same velocity as the belt and the friction force can have any value within its bounds; the friction force is not known but the acceleration of the block is known as well as its position (the same as the point of the belt the mass is stuck). The dynamical equation acts as a constraint to find the unknown friction force. In the slip part of the motion one has a standard dynamical problem, solved by integration of the dynamical equation. In the phase plane, while the slip region has non-zero measure, the stick region has zero measure and is a manifold of lower dimension, one or zero.

The focus of the paper is to quantify the percent of time in which the block stays in the stick-mode for different models of periodic belt velocity, a sort of biased-coin game: stick or slip. The percent of time gives the probability in a Bernoulli trial. Numerical experiments were carried on considering continuous, discontinuous, and random models of belt velocity. For each model, the duration of the stick-mode was computed as a function of friction coefficient, μ, and as a function of ratio, r, between the frequency of the belt velocity and the natural frequency of the system. The results show that varying the system configuration the manifold of possibilities for the stick–slip distribution is very rich. Depending on the model of belt velocity, on μ, and on r, the stick-mode can take a large fraction of the duration of the solution, reaching values up to 100 percent, or may even disappear. It is important to remark that as deterministic and stochastic models of belt velocity were considered, the analysis of the duration of the stick-mode was done from a deterministic and also from a stochastic viewpoint.

This paper is organized as follows. Section 2 describes a most simple stick–slip oscillator. It presents also the analytical solutions of the system response and an energy analysis of the system for the case of belt with constant velocity. In this energy analysis, the phase diagram of the system response is divided into regions and for each one of them it is determined if the friction force introduces or dissipates energy. In Section 3, two deterministic and periodic models of belt velocity are considered. Their influence in the system behavior is analyzed numerically through the phase diagram in which the stick parts of the trajectory are highlighted. Section 4 presents results of the stick duration as a function of μ and r for each deterministic model of belt velocity considered in Section 3. The construction of the probabilistic model of the uncertain belt velocity is given in Section 5. Statistics of the duration of the stick-mode, as mean value and confidence intervals, are presented in Section 6.

2. Dynamics of the stick–slip oscillator

As explained in the introduction, the system analyzed in this paper is composed of a block connected to a fixed support by a spring. The block moves over a continuous belt that is driven by rollers, as shown in Fig. 1. It is assumed that there is friction between the block and the belt which is modeled as a Coulomb friction. Due to this friction model, the resulting motion of the block can be characterized in two qualitatively different modes with a non-smooth transition between them. The first one is the stick-mode, in which the block and belt have the same velocity during an open time interval. The second one is the slip-mode, in which they have different velocities. The position of the block over the belt is represented by x and its equation of motion is

$$m\,\ddot{x}(t)+k\,x(t)=f(t), \tag{1}$$