



Wave propagation in one-dimensional waveguides with slowly varying random spatially correlated variability

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ABSTRACT

This paper investigates structural wave propagation in waveguides with randomly varying material and geometrical properties along the axis of propagation. More specifically, it is assumed that the properties vary slowly enough such that there is no or negligible backscattering due to any changes in the propagation medium. This variability plays a significant role in the so called mid-frequency region for dynamics and vibration, but wave-based methods are typically only applicable to homogeneous and uniform waveguides. The WKB approximation is used to find a suitable generalization of the wave solutions for finite waveguides undergoing longitudinal and flexural motion. An alternative wave formulation approximation with piecewise constant properties is also derived and included, so that the internal reflections are taken into account, but this requires a discretization of the waveguide. Moreover, a Fourier like series, the Karhunen–Loeve expansion, is used to represent homogeneous and spatially correlated randomness and subsequently the wave propagation approach allows the statistics of the natural frequencies and the forced response to be derived. Experimental validation is presented using a cantilever beam whose mass per unit length is randomized by adding small discrete masses to an otherwise uniform beam. It is shown how the correlation length of the random material properties affects the natural frequency statistics and comparison with the predictions using the WKB approach is given.

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1. Introduction

Manufacturing processes often result in variability of properties compared to the nominal designed product. As the requirements for optimum design increase and include a broader frequency range for the dynamic response, it is important to improve prediction capabilities. Element based techniques, such as the Finite Element (FE) method [1,2], are the main prediction tools for structural dynamics in industrial applications and are typically used for deterministic predictions. However, the higher the frequency range under analysis using FE, the finer the mesh requirement, with increased computational cost. The pollution effect, i.e. when the accuracy of the FE solution degenerates as the wavenumber or frequency increases [3], must also be taken into account, further increasing the computational cost. Besides, even small

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variability starts to play an important role such that a deterministic FE approach by itself is no longer able to predict the structure's behaviour accurately. It is then necessary to add some level of randomness within the description of the structural models and therefore increasing the computational cost further.

In this context, three different frequency bands of analysis can be defined for the typical dynamic response of weakly dissipative structures [4,5]: the low-frequency range, where deterministic models are well suited, with low order modes and low sensitivity to variability, and FE-like approaches are applicable; the high-frequency range, where energy-like methods (for instance, Statistical Energy Analysis – SEA [6]) are well suited, with many high order modes involved; and the mid-frequency range, where individual modal behaviour is still distinguishable and affected by variability. The latter frequency range is too high in frequency to be efficiently treated using FE methods, but not high enough for energy-based methods to be applicable.

Wave-based methods have been developed in order to attempt to bridge this gap in the prediction tools, by increasing computational efficiency, and therefore extend the applicability of deterministic models to higher frequencies. However, most of them assume that waveguide properties are homogeneous in the direction of the travelling wave, limiting the application of such approaches. Examples include the Wave Based Method (WBM) [5,7,8], based on the indirect Trefftz approach, the spectral element method [9,10], that uses analytical solutions for the wave propagation to assemble dynamic stiffness matrices for waveguides, the Semi-Analytical FE method [11], that uses a FE formulation for the cross section of waveguides and assumes a wave like solution in the direction of propagation of the waves, and the Wave and Finite Element (WFE) method [12–14], that applies the theory of periodic structures for homogeneous waveguides using a FE model of the cross section.

Limitations in the application of these wave approaches arise because analytical solutions for non-homogeneous waveguides are only possible for very particular cases, for example acoustic horns, ducts, rods and beams – e.g. [15–17] – particularly when the spatial variation in the properties of the waveguide with respect to distance compared to the wavelength are small. Lee et al. [18] have shown that the velocity of energy propagation is different to the group velocity for a class of deterministic non-homogeneity in one-dimensional waveguides, such that no wave conversion occurs. Langley [19] has shown, using a perturbation approach, that the amplitude of a wave travelling along a non-homogeneous one dimensional waveguide changes and that power is conserved. Also, he addressed the issue of wave reflection at a cut-on section. Moreover, Scott [20] has also considered the statistics of the wave intensity and phase for a specific class of one dimensional random waveguides, also considering negligible scattering.

Randomness in waveguides has also been considered in the context of SEA. Manohar and Keane [21] derived expressions for the probability density functions (PDFs) of the natural frequencies and mode shapes for a class of stochastic rods, given that the mass density is a Gaussian random field and that a specific relation with the axial stiffness exists such that there is only phase change. Subsequently the flow of energy between coupled stochastic rods was considered [22]. Moreover, a numerical approach, using the WFE method, has been proposed by Ichchou [23] to include spatially homogeneous variability in waveguides using a first order perturbation [24,25].

Material or geometrical uncertainty often exhibits spatial correlation, particularly when dealing with composite materials [26–29], and the theory of random fields, being multidimensional random processes, can be used to model this spatially distributed variability using a probability measure [30,31]. A solution of the physical problem might typically require the variability to be incorporated within a spatial discretization, e.g. an FE model, of the physical domain [32]. A number of discretization methods are available, among them being the midpoint method. This was first introduced by Der Kiureghian [33] and is used in this study. It consists in approximating the random field in a spatial domain, previously discretized by a given mesh, by using a constant but random value within each element, or group of elements. This value is given by a sample of the random field specified at the geometric centre of the element. This approach is very appealing, because it does not require any modifications of the FE code and hence making it suitable for using with commercial FE in a framework of Monte Carlo sampling.

In addition, a Karhunen–Loeve (KL) expansion [31] provides characterization of the random field in terms of deterministic eigenfunctions weighted by uncorrelated random variables. For some families of correlation functions and specific geometries, there exist analytical expressions for the KL expansion, but, in general, a numerical approach has to be used [34]. The KL expansion allows a significant reduction in the number of random variables required for a random field simulation, since the number of KL modes retained in the analysis N_{KL} is usually much smaller than the number of elements in the mesh. Moreover the expansion is optimal in a mean square sense [31].

In this work, the WKB (after Wentzel, Kramers and Brillouin) approximation for the wave solution is used along with a generalized approach for one-dimensional waveguides in terms of propagation, reflection and transmission matrices, as described by Harland et al. [35]. This approximation requires the internal reflections to be negligible or not to occur due to any local changes in the material or geometrical properties. This approach is subsequently used herein to derive expressions for the natural frequencies and input mobility of finite length waveguides. It is also shown that random fields can be straightforwardly included in the formulation, and an analytical expression of the KL expansion is used to derive closed form approximations for the PDF of the natural frequencies of straight rods and beams.

The key assumption made in the WKB approximation is that there is negligible backscattering of the propagating wave due to changes in local properties. A more general approximation can be achieved if one considers a waveguide with piecewise constant material variability, separated into a finite number of discrete sections. This is an approximate representation for the more general spatially varying system. In this way, internal reflections due to local changes in the impedance can be taken into account at the junctions of the sections. A more general analysis of this kind of system can be found in [36,37].

In the next section, the WKB approximation is reviewed and the main results for free and forced vibration response given for rod and beam examples. A formulation in terms of propagation, reflection and scattering matrices is presented in detail

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