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In-plane vibrations of a rectangular plate: Plane wave expansion modelling and experiment

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A B S T R A C T

Theoretical and experimental results for in-plane vibrations of a uniform rectangular plate with free boundary conditions are obtained. The experimental setup uses electromagnetic-acoustic transducers and a vector network analyzer. The theoretical calculations were obtained using the plane wave expansion method applied to the in-plane thin plate vibration theory. The agreement between theory and experiment is excellent for the lower 95 modes covering a very wide frequency range from DC to 20 kHz. Some measured normal-mode wave amplitudes were compared with the theoretical predictions; very good agreement was observed. The excellent agreement of the classical theory of in-plane vibrations confirms its reliability up to very high frequencies

1. Introduction

There is an increasing interest in the in-plane vibrations of plates. This is due to the fact that, in certain specialized applications, high-frequency vibrations appear. This commonly happens in data storage systems, in the kHz range, in which in-plane vibrations cause a problem in following narrow data tracks [1]. These vibrations are also important in ship hull design since there is evidence that in-plane vibrations and high-frequency noise are strongly related [2]. The in-plane modes also play an important role in the transmission of high frequency vibrations through a built-up structure [3]. Furthermore, in-plane modes can be used for non-destructive testing and evaluation of elastic constants [4]. Finally, as in-plane vibrations appear at higher frequencies than transverse vibrations, finite element calculations are more difficult for the former. All these, and other problems not listed, have led to a renewed interest in the phenomenon of in-plane vibration of rectangular plates that cover several orders of magnitude from nanosystems to macrostrutures.

There are several recent significant theoretical and numerical contributions to the study of the in-plane vibrations of plates [1–3,5–12]. However, experimental results have been, until recently, very scarce [4,13]. There are two likely reasons of this fact: first, in-plane vibrations appear at high frequencies and second, the measurement of transverse vibrations is easier than the excitation and detection of in-plane modes [13–16]. Such state of affairs started to change when electronic speckle pattern interferometry — ESPI or TV holography — made possible the measurement of in-plane modes [14,17,18]. Thus, there

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is the need for research on the in-plane vibrations of plates to consolidate the classical theory of in-plane vibrations, especially at high frequencies, to contrast such theory with experimental results.

This work adds to the literature on the subject of in-plane vibration of plates which can be found in Ref. [11]. In the next section, the plane wave expansion method, applied to the classical theory of in-plane vibrations of thin plates, is introduced. This method will be used to calculate the normal modes of a rectangular plate with two sets of boundary conditions, namely all its boundaries free (F–F–F–F) and all edges clamped (C–C–C–C). In Section 3, the experimental methodology to measure the in-plane vibrations of a rectangular plate with free boundaries, using electromagnetic-acoustic transducers and a vector network analyzer, is presented. In Section 4 the theoretical normal-mode frequencies and wave amplitudes are compared with the experimental results, showing a very good agreement. Finally, in Section 5, some conclusions are given.

2. The plane wave expansion method for the in-plane wave equation

The plane wave expansion (PWE) method refers to a computational technique to solve partial differential equations as an eigenvalue problem [19]. This method is popular among the photonic (phononic) crystal community to obtain the dispersion relation of artificial crystals [20–23]. In a previous work [16] it was shown that the PWE method can be implemented to solve the out-of-plane Kirchhoff–Love equation for finite systems. As shown below, this numerical method is also useful to solve the in-plane wave equation for finite systems. The main difference between the plane wave expansion method and other numerical methods is that the boundary conditions are not imposed but are rather simulated by introducing a second medium with certain physical properties. A rectangular cell (see Fig. 1) of dimensions $a\times b$ will be used. The plate is located at the center of the cell surrounded by a host material that, for a plate with free ends, mimics the vacuum and, for a plate with clamped ends, mimics an extremely hard medium [16]. The unit cell is repeated periodically in both directions and its mechanical parameters are replaced by a Fourier series truncated at N plane waves. In what follows, the PWE method, as used to calculate the in-plane normal modes of plates with free-ends, will be described in detail.

The equations that govern the in-plane motion, in the classical theory of in-plane waves of a thin plate, are [24]

$$\frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=\rho h\frac{\partial^2 u}{\partial t^2},$$

$$\frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}=\rho h\frac{\partial^2 v}{\partial t^2}, \tag{1}$$

where h and ρ are the thickness and the density of the plate, respectively. The variables $u(x,y)$ and $v(x,y)$ are the displacements in the X and Y directions, respectively, while the plate stresses are

$$N_x=C(e_{xx}+\nu e_{yy}),$$

$$N_y=C(e_{yy}+\nu e_{xx}),$$

$$N_{xy}=C(1-\nu)e_{xy}, \tag{2}$$

where ν is Poisson's ratio and C is the extensional rigidity given by

$$C=\frac{Eh}{1-\nu^2}, \tag{3}$$

with E standing for Young's modulus. The strain–displacement relations are

$$e_{xx}=\frac{\partial u}{\partial x},\quad e_{yy}=\frac{\partial v}{\partial y}\quad\text{and}\quad e_{xy}=\frac{1}{2}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right). \tag{4}$$

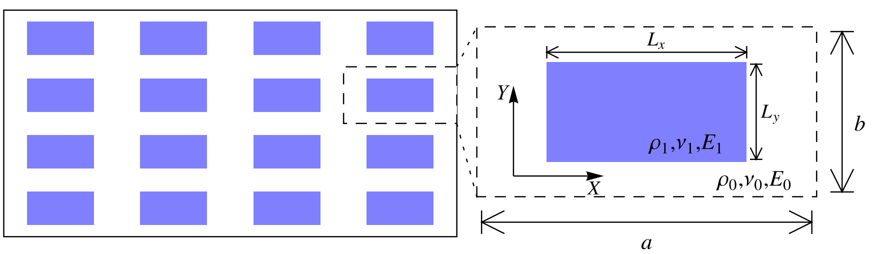

Fig. 1. Plane wave expansion method: a rectangular cell of sides a and b is repeated periodically on the plane. Each cell is composed of the plate of interest, in the center of the cell, surrounded by a host material. The elastic properties of the plate of interest have the subindex 1 while the elastic constants of the host material have the subindex 0. The host material mimics the vacuum, in a certain limit, that yields the free–end boundary conditions for the inner plate.