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Structural source identification using a generalized Tikhonov regularization

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ABSTRACT

This paper addresses the problem of identifying mechanical exciting forces from vibration measurements. The proposed approach is based on a generalized Tikhonov regularization that allows taking into account prior information on the measurement noise as well as on the main characteristics of sources to identify like its sparsity or regularity. To solve such a regularization problem efficiently, a Generalized Iteratively Reweighted Least-Squares (GIRLS) algorithm is introduced. Proposed numerical and experimental validations reveal the crucial role of prior information in the quality of the source identification and the performance of the GIRLS algorithm.

1. Introduction

The control of the vibro-acoustic behavior of structures remains a challenging task in many industrial applications. A possible solution is to control vibration at sources. In this situation, the knowledge of excitation sources is required. However, such information is sometimes difficult or even impossible to measure. A possible alternative to bypass this difficulty is to identify excitation sources from vibration measurements. The resolution of this inverse problem has been extensively studied over the past three decades. The most widespread approaches are based on the transfer functions matrix of the structure under test [1–5], that can be numerically computed [2,6] or measured [7–9]. There exist, however, alternative approaches based on the numerical calculation of a local operator corresponding to the dynamic stiffness of a part of the structure [10–14].

Unfortunately, all the proposed methods are very sensitive to measurement errors causing the identification to fail. That is why, there have been numerous studies focusing on the development of regularization methods to stabilize this inverse problem. Except methods for which very specific procedures have been proposed [10,15], the most popular regularization methods remain the Truncated Singular Value Decomposition (TSVD) [16–20] and the Tikhonov regularization [6,13,14,20–22].

In the present paper, we are interested in solving the identification problem using the following generalized Tikhonov regularization [23]:

$$\widehat{\mathbf{F}}_{\mathbf{c}} = \arg\min_{\mathbf{F}_{\mathbf{c}}} \frac{1}{p}\|\mathbf{H}\mathbf{F}_{\mathbf{c}} - \mathbf{X}_{\mathbf{m}}\|_p^p + \frac{\lambda}{q}\|\mathbf{L}\mathbf{F}_{\mathbf{c}}\|_q^q, \quad \forall (p,q) \in \,]0,\infty[^2, \tag{1}$$

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where $\mathbf{X_m}$ is the vibration field (displacement, velocity or acceleration) measured over the structure, $\mathbf{F_c}$ is the force vector to identify, $\mathbf{H}$ is the corresponding transfer functions matrix, $\|\bullet\|_p$ is either the l_p norm if $p \geq 1$ or the l_p quasi-norm if $p < 1$ and $\widehat{\mathbf{F}}_\mathbf{c}$ is the particular value of $\mathbf{F_c}$ for which the functional reaches its minimum. Such a problem is said to be convex when $p \geq 1$ and $q \geq 1$ and non-convex otherwise. In the next of the paper, the proposed generalized Tikhonov regularization is also referred to as l_p–l_q regularization.

In Eq. (1), the functional to minimize is the sum of two terms. The first one is the data fidelity term $(1/p)\|\mathbf{HF_c}-\mathbf{X_m}\|_p^p$, which is related to measurement noise and allows controlling the a priori on the nature of the noise [24,25]. The second one is the regularization term $(1/q)\|\mathbf{LF_c}\|_q^q$, that introduces an a priori on the solution, like its regularity thanks to the differentiation matrix $\mathbf{L}$ [26], or its sparsity, by using an l_q norm such as $q < 2$ [27]. Finally, these terms are related by the trade-off parameter, $\lambda \in \mathbb{R}^+$, that represents the balance between data fidelity and regularization terms. In this formalism, the standard Tikhonov regularization corresponds to $p=q=2$. As shown in [13,14], this regularization leads to a systematic smoothing of regularized solutions, which is not a desirable effect when one wants to identify localized sources, for instance.

To have a better understanding of this result, it is worth turning to deconvolution techniques developed in image and signal processing, in which the influence of data fidelity and regularization terms has been deeply studied. It has been shown in [25] that having a data fidelity term reflecting the noise characteristics of the signal provides better reconstruction. In particular, it has been pointed out that using the l_2 norm for the data fidelity term was most appropriate to remove additive Gaussian white noise [24,28], whereas using l_1 norm was suitable for removing impulsive noise [28,29]. Concerning the regularization term, it has been proven that sparsity-promoting terms allow preserving discontinuities in deconvolved signal or image [30–34]. The most widely used are the regularization terms based on the l_1 norm [30,31]. However, it has been demonstrated recently that non-convex sparse regularization terms, i.e. based on the l_q quasi-norm ($q \in]0,1[$), yield even better discontinuities preservation when compared to the convex l_1 type regularization [35–37]. To solve efficiently convex as well as non-convex sparse regularization problems, only a few algorithms are available. They are generally based on a generalized version of the Iteratively Reweighted Least-Squares (IRLS) algorithm [38–40], originally proposed to deal with problems of the form of Eq. (1) for $\lambda=0$ [41–43].

To the author's knowledge, the aforementioned methods have been seldom applied in the context of structural source identification. One can nevertheless cite the recent work of Chardon and Daudet [44], in which they apply a convex l_2–l_1 regularization, processed by a group matching pursuit algorithm, to localize sources acting on a thin plate. Finally, it is important to cite works of Guillaume et al. [8,9] that propose using an IRLS type procedure to identify localized sources, as well as the work of Renzi et al. [13,45], in which they propose to apply a Richardson–Lucy deconvolution to address, at a post-process stage, the setback of the smoothing effect of Tikhonov regularization when identifying localized sources.

This paper proposes demonstrating the ability of the generalized Tikhonov regularization to identify vibration sources using a GIRLS algorithm as a solver. Unlike the standard IRLS procedure, the proposed approach allows introducing, besides prior information on measurement noise, an a priori on the spatial distribution of sources.

To clearly distinguish the main features of the proposed regularization procedure, the present paper is divided into four parts. In Section 2, the general methodology, based on a Finite Element (FE) modeling of a part of the structure under test, is presented. In Section 3, the GIRLS algorithm is detailed. A particular attention is paid to the crucial choice of the regularization parameters and the definition of a robust stopping criterion. Finally, Sections 4 and 5 are devoted to the numerical and experimental validations of the proposed methodology respectively. Obtained results show that using properly a priori knowledges on the measurement noise and the spatial distribution of the solution improves drastically the quality of the identification and the performance of the GIRLS algorithm.

2. General principles of the identification process

As explained in the introduction, one seeks to deal with the structural source identification problem using the generalized Tikhonov regularization given by Eq. (1). To properly solve this problem, one needs to know the mechanical behavior of the part of the structure under test. Such information is given by the transfer functions matrix $\mathbf{H}$. Since the measurement of $\mathbf{H}$ can be quite cumbersome, one prefers using a FE model of the structure instead.

Assuming the structure is linear and the damping is of structural type, the FE model of the structure in the frequency domain is of the form:

$$[\mathbf{K}(1+j\eta)-\omega^2\mathbf{M}]\mathbf{X}(\omega)=\mathbf{D}(\omega)\mathbf{X}(\omega)=\mathbf{F}(\omega), \tag{2}$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{K}$ is the stiffness matrix, $\mathbf{X}(\omega)$ is the vector containing the considered degrees of freedom (dofs) of the structure, $\mathbf{F}(\omega)$ is the excitation vector, η is the structural damping factor, $\mathbf{D}$ (ω) is the dynamic stiffness matrix of the structure and ω is the angular frequency.

In Eq. (2), the vector $\mathbf{X}(\omega)$ contains two types of degrees of freedom (dofs), namely translation and rotation dofs. However, in practical situations, some dofs, such as rotations, are difficult to measure directly. Considering this experimental fact, the FE model given by Eq. (2) has to be transformed into a model containing only the measurable dofs. Several techniques have been used for that purpose, such as static condensation [21], observation matrix approach [14,46] or exact dynamic condensation [11,45,47,48]. Finally, reduction techniques can be used to avoid the need of data in unmeasured areas, such as the Craig–Bampton reduction [13,45,49] or the modal reduction [50].