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Multi-frequency excitation of stiffened triangular plates for large amplitude oscillations

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ABSTRACT

Free and forced vibrations of triangular plate are investigated. Diverse types of stiffeners were attached onto the plate to suppress the undesirable large-amplitude oscillations. The governing equation of motion for a triangular plate, based on the von Kármán theory, is developed and the nonlinear ordinary differential equation of the system using Galerkin approach is obtained. Closed-form expressions for the free undamped and large-amplitude vibration of an orthotropic triangular elastic plate are presented using the two well-known analytical methods, namely, the energy balance method and the variational approach. The frequency responses in the closed-form are presented and their sensitivities with respect to the initial amplitudes are studied. An error analysis is performed and the vibration behavior, as well as the accuracy of the solution methods, is evaluated. Different types of the stiffened triangular plates are considered in order to cover a wide range of practical applications. Numerical simulations are carried out and the validity of the solution procedure is explored. It is demonstrated that the two methods of energy balance and variational approach have been quite straightforward and reliable techniques to solve those nonlinear differential equations. Subsequently, due to the importance of multiple resonant responses in engineering design, multi-frequency excitations are considered. It is assumed that three periodic forces are applied to the plate in three specific positions. The multiple time scaling method is utilized to obtain approximate solutions for the frequency resonance cases. Influences of different parameters, namely, the position of applied forces, geometry and the number of stiffeners on the frequency response of the triangular plates are examined.

1. Introduction

Vibration analyses of elastic plates with different geometries received much attention due to various applications in mechanical, civil and aerospace engineering fields. Comprehensive technical articles focused on free and forced vibrations and stability of elastic plates have been published [1–4].

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Triangular plates are often used as essential components in many engineering applications, such as aircraft wings, railway and road vehicle bodies, ship decks, skew bridges, floor slabs and ceiling walls. Consequently, many articles have so far been published on the dynamic analysis of rectangular and triangular plates. Analytical solutions of plates do not exist in general and hence, most researchers have used either approximate techniques or numerical methods to obtain the natural frequencies of plates. Kim and Dickinson [5,6] employed the Rayleigh–Ritz approach to analyze the free vibration of thin triangular plates with different boundary conditions.

The natural frequencies and corresponding mode shapes of free vibration of triangular plates were determined using the Ritz method [7] and the free vibration of partially supported triangular plates was analyzed numerically using finite element method [8]. Free vibration analysis of thick cantilevered triangular plates, using the Rayleigh–Ritz method, was done [9] and results of free vibrations of a triangular plate with variable thickness, based on Green function of an equivalent rectangular plate, were reported [10].

Dynamic characteristics of a general type of triangular plate with variable thickness, using the Rayleigh–Ritz method, were published [11]. Natural vibration characteristics of triangular plates with partially clamped end conditions were studied experimentally [12]. An innovative method, based on the Green function of an equivalent rectangular plate, was proposed to obtain the numerical values of the natural frequencies of an orthotropic triangular plate [13]. The triangular differential quadrature method was used by Zhong [14] for the free flexural vibration analysis of isosceles triangular Mindlin plates. The Ritz method was used to study the vibration of a skewed cantilevered triangular plate numerically [15]. Free vibration of a general form of triangular composite plates with elastically restrained edges was investigated using the Rayleigh–Ritz method [16]. The *h–p* finite element method was used to obtain the accurate values of natural frequencies of a triangular orthotropic plate [17,18].

Reviewing the reported articles in this area indicate that most of the works are dealing with linear theories and small deformations due to their simplifications of the analyses. These assumptions are no longer valid when the weight of structures is comprehensively reduced and plate elements are subjected to deflections with considerable magnitude. Large-amplitude vibration of either orthotropic or isotropic triangular plates has not been analytically investigated. Analytical solutions are always preferred over numerical ones since they can provide far better insight to structure and also improve engineering design judgment.

The main objectives of the present work are: (i) to develop the governing equation of orthotropic triangular plates based on the von Kármán theory, (ii) to present an analytical frequency–amplitude relationship for large-amplitude undamped free vibration of a triangular plate, (iii) to investigate the case of multi-frequency excitation of triangular orthotropic plates, and (iv) to set forth with accuracy as how the vibration levels of structure is suppressed using different types of stiffeners.

Because of complexities usually arise with the solution of nonlinear differential equations; many of the non-perturbative approaches have been recently developed for that purpose. The homotopy perturbation method, variational approach, energy balance method, frequency–amplitude formulation and homotopy analysis method are few examples for such recently developed techniques used in varieties of nonlinear applications [19–25].

The method of Energy Balance, which is used in this research, has been employed by other researchers to find the analytical solution and the nonlinear natural frequency of different types of nonlinear conservative oscillators. Examples of these are the Generalized Duffing equation [20], Helmholtz Duffing equation [21] and the nonlinear oscillator with fractional power [22]. The Variational approach, which is used in this study, has been utilized by other investigators to determine the natural frequency of many nonlinear systems. It has been used to find the analytical solutions and the nonlinear natural frequency of Generalized Duffing equation [26] and the nonlinear oscillator with fractional power [27].

The multiple time scales method is a very potent and well-known approach, which has been used by several researchers, to investigate the natural frequency and also the primary, higher-order, internal, super-harmonic and sub-harmonic resonances [28–30,33,34]. Furthermore, several studies have been carried out to investigate the multi-frequency excitations in different types of structures using the multiple time scale method [28]. Combination resonances in the response of Duffing oscillator to a three-frequency excitation can be mentioned as one of the earliest efforts in this field [28].

Response of a nonlinear system with strong damping to multi-frequency excitations was analyzed and a new asymptotic calculation method was presented for a class of nonlinear non-autonomous vibration systems with multiple external periodic interferences [29]. Resonances in nonlinear structures, subjected to multi-frequency excitation, are studied and response of single-degree-of-freedom system with quadratic, cubic and quartic nonlinearities subjected to sinusoidal excitation that involves multiple frequencies is obtained [30].

The multiple time scaling method was used to construct a first-order uniform expansion yielding to two first-order nonlinear ordinary differential equations that were derived to determine the amplitude and phase. These oscillations involved sub-harmonic oscillation of order one-fourth and a super-harmonic oscillation of order two. The steady-state responses and their stability analyses were computed for selected values of the system parameters [30].

The analysis of large-amplitude oscillations of triangular orthotropic plates, based on the von Kármán equations, is presented in this study. Three different potent analytical approaches, namely, the energy balance method (EBM), variational approach (VA), and the multiple time scaling method (MTM) are employed to investigate the free and forced vibrations of the system. The frequency–amplitude relationships are obtained analytically for large-amplitude of vibration. Stiffened plates are considered as the practical cases for the numerical simulation studies. A parametric sensitivity analysis is carried out and the effects of different parameters on the frequency–amplitude responses, obtained from the EBM and VA, are