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Analytical modeling and vibration analysis of internally cracked rectangular plates

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ABSTRACT

This study proposes an analytical model for nonlinear vibrations in a cracked rectangular isotropic plate containing a single and two perpendicular internal cracks located at the center of the plate. The two cracks are in the form of continuous line with each parallel to one of the edges of the plate. The equation of motion for isotropic cracked plate, based on classical plate theory is modified to accommodate the effect of internal cracks using the Line Spring Model. Berger's formulation for in-plane forces makes the model nonlinear. Galerkin's method used with three different boundary conditions transforms the equation into time dependent modal functions. The natural frequencies of the cracked plate are calculated for various crack lengths in case of a single crack and for various crack length ratio for the two cracks. The effect of the location of the part through crack(s) along the thickness of the plate on natural frequencies is studied considering appropriate crack compliance coefficients. It is thus deduced that the natural frequencies are maximally affected when the crack(s) are internal crack(s) symmetric about the mid-plane of the plate and are minimally affected when the crack(s) are surface crack(s), for all the three boundary conditions considered. It is also shown that crack parallel to the longer side of the plate affect the vibration characteristics more as compared to crack parallel to the shorter side. Further the application of method of multiple scales gives the nonlinear amplitudes for different aspect ratios of the cracked plate. The analytical results obtained for surface crack(s) are also assessed with FEM results. The FEM formulation is carried out in ANSYS.

1. Introduction

Plate as basic structural element, finds application in many fields such as mechanical, civil, aerospace, ship building, bunkers, reservoirs etc. The vibration characteristics of rectangular plates are of interest especially in the areas of mechanics and aerospace engineering. Many structural systems such as aircraft wings, vehicle parts, and helicopter blades are routinely modeled as beams and plates. The presence of singularities in the form of crack(s) and hole(s) affect the vibration characteristics of plates. The literature on vibrations of cracked plate is reviewed with a motivation to develop analytical model of equation of motion of cracked plate. Gorman [1] studied the free vibrations of cantilever plates by method of superposition. Narita [2] studied the free vibration behavior of anisotropic rectangular plates for combinations of general

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edge conditions. Cheng et al. [3] applied green's functions for infinite and semi-infinite anisotropic thin plates. Wu et al. [4] studied exact solutions for free vibration analysis of rectangular plates using Bessel functions. Okamura et al.[5] examined the effect of crack on lateral deflection and load carrying capacity of columns considering crack compliance due to bending only. Rice and Levy [6] formulated the line spring model using classical plate theory, wherein the crack is represented by continuously distributed line spring with stretching and bending compliances. The authors matched the crack compliance coefficients with those for an edge cracked strip under plane strain. They also established the variation of tensile and bending compliance with crack depth. King [7] converted the coupled integral equations given by Rice and Levy [6] to a set of linear algebraic equations to analyze fracture parameters thus simplifying the line spring model. Zheng and Dai [8] then applied this simplified line spring model to find stress intensity factors in their analytical modeling of a rectangular plate with angled crack. Khadem and Rezaee [9] introduced modified comparison functions for vibration analysis of cracked rectangular plate considering bending compliance. The authors developed an analytical method for cracked plates and established that the presence of a crack at a specific depth and location affects the natural frequencies differently. Stahl [10] analyzed vibration and stability of cracked rectangular plates. Solecki [11] solved the problem of bending vibration of a simply supported rectangular plate with a crack parallel to one edge. Qian et al.[12] devised a finite element model for vibration of cracked plates. Krawczuk [13] developed a finite element scheme for plate with elastic-plastic through crack. Chen and Shen [14] applied finite element alternating method to bending of cracked plate with mixed mode cracks. Maruyama and Ichinomiya [15] in their experimentation on clamped rectangular plates considered various lengths, inclinations and locations of narrow slits. They established variation of natural frequency with the angular orientation of slits. Wu and Law [16] experimentally concluded that the orientation of the crack in a thick plate affects the frequency for free boundary condition. The literature does not appear to contain substantial research on analytical model of cracked rectangular plate until Israr et al.[17,18] developed an approximate analytical model for vibrations of cracked plate using line spring model to represent bending and tensile forces at crack location wherein, the surface crack is parallel to one of the edges of the plate and is located at the center. They considered three different boundary conditions of support and employed a perturbation technique called method of multiple scales to obtain amplitude response. Their work is based on classical plate theory and the line spring model [6] for finding stress intensity factors of cracked plate is used therein to represent relationship between tensile and bending stress at far sides of the plate, and at crack location. They calculated compliance coefficients for bending and tension considering crack depth to plate thickness ratio of 0.6. It can be concluded from the work of Israr et al. [17,18] that the natural frequencies go on decreasing and peak amplitude increases as the crack length increases. Wu and Shih [19] obtained nonlinear response of cracked plate applied with periodic in-plane load and concluded that the response of the system depends on crack location and aspect ratio of the plate. Irwin [20] calculated crack extension force for a part through crack in a plate considering the strain energy of the net ligament. Huang and Leissa [21] introduced special displacement function in the Ritz method for free vibration analysis of rectangular plates with side cracks. Recently Huang et al. [22] established a set of new admissible functions for the Ritz method in free vibration analysis of simply supported square plate. They considered arbitrary position and various angular orientation of a through internal crack. More recently, Ismail and Cartmell [23] developed an analytical model for vibration analysis of a cracked rectangular plate considering various angular orientation of the crack, extending the work of Ref. [17]. The authors established relations for moment and in-plane force due to orientation of the crack and validated their findings with experimentation. Extending their own work, Ismail and Cartmell [24] validated their analytical model with finite element model for natural frequencies and vibration amplitude. The literature shows that the nonlinear vibration characteristics of cracked plate depend on parameters like crack length, orientation, plate geometry, damping and location of the crack. Thus it is instructive to extend the currently developing field of vibration analysis of cracked plate when the crack is internal.

2. Problem description and methodology

The present work references the analytical model proposed by Israr et al. [17], extended by Ismail and Cartmell [23] and applies it for the case of a rectangular plate containing two perpendicular surface cracks and develops a novel analytical model when the single and the two cracks are internal as shown in Fig. 1(a) to (c). L_1 and L_2 are plate dimensions along x and y direction respectively, d is the offset distance between crack center line and neutral plane of the plate. The plate thickness is h, 2a and 2b are the crack lengths parallel to x and y axis respectively. The crack(s) are in the form of continuous line and the effect of rotary inertia and through thickness shear is neglected. The equation of motion becomes nonlinear by addition of in-plane forces according to Berger's formulation [17]. Crack terms are formulated by appropriate compliance coefficients using line spring model and the relationship between nominal tensile and bending stress at the far sides of the plate and at the crack location is applied for the two cracks. Three boundary conditions namely (i) All edges simply supported (SSSS) (ii) Two adjacent edges clamped and two free (CCFF) and (iii) Two adjacent edges clamped and two simply supported (CCSS) are considered to analyze the effect of internal cracks on the natural frequencies of vibration. The results of variation of natural frequencies and amplitudes with length ratio of the two cracks are presented. Furthermore the effect of location of single and the two cracks along the thickness of plate on natural frequencies is studied by using appropriate crack compliances for bending and stretching.

The equation of motion governing the nonlinear vibrations of an isotropic rectangular plate with two perpendicular part-through surface cracks located at the center of the plate is derived following the equilibrium principle in order to get a tractable solution. The two cracks are in the form of continuous lines, each crack is parallel to one of the edges of the plate as