Journal of Sound and Vibration 333 (2014) 6858–6868

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Inverse problem with beamforming regularization matrix applied to sound source localization in closed wind-tunnel using microphone array

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ARTICLE INFO

Article history:
Received 18 December 2013
Received in revised form
21 May 2014
Accepted 23 July 2014
Handling Editor: P. Joseph
Available online 2 September 2014

ABSTRACT

Microphone arrays have become a standard technique to localize and quantify source in aeroacoustics. The simplest approach is the beamforming that provides noise source maps with large main lobe and strong side lobes at low frequency. Since a decade, the focus is set on deconvolution techniques such as DAMAS or Clean-SC. While the source map is clearly improved, these methods require a large computation time. In this paper, we propose a sound source localization technique based on an inverse problem with beamforming regularization matrix called Hybrid Method. With synthetic data, we show that the side lobes are removed and the main lobe is narrower. Moreover, if the sound noise source map provided by this method is used as input in the DAMAS process, the number of DAMAS iterations is highly reduced. The Hybrid Method is applied to experimental data obtained in a closed wind-tunnel. In both cases of acoustic or aeroacoustic data, the source is correctly detected. The proposed Hybrid Method is found simple to implement and the computation time is low if the number of scan points is reasonable.

1. Introduction

Over the last two decades, phased-microphone arrays have become a standard technique to localize aeroacoustic sources [1,2]. Beamforming is a basic approach that consists in delaying and summing microphone signals. The advantage of beamforming is its simplicity and robustness. One of the main problems in beamforming is the poor spatial resolution at low frequency which somehow limits the interpretation and relevance of source maps.

To overcome this issue, techniques have been developed in the past years. One of them is a deconvolution technique called DAMAS [3] (that has been extended to DAMAS2 [4] and DAMAS-C [5]). The aim of DAMAS is to extract a source distribution from a beamforming source map by iteratively deconvolving the map. The width of the main lobe is clearly reduced with this technique and side lobes are suppressed. However, the computational cost is high and uncorrelated monopoles have to be used as the reference solutions. An alternative approach, called Clean-SC [6], has been developed. The aim is to suppress side lobes correlated with the main source. Therefore, Clean-SC cannot discriminate coherent sources. Recently, an algorithm has been developed by Suzuki [7]. After decomposing the microphone Cross Spectral Matrix (CSM) into eigenmodes, an inverse problem is defined and solved iteratively. The spatial resolution of this approach is increased

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http://dx.doi.org/10.1016/j.jsv.2014.07.028
0022-460X/

but the computation time is large due to the iterative nature of the method. New methods based on sparsity constraint [8] or Bayesian approach [9] allow super-resolution in acoustic imaging, but the computation time is very large and these methods use dedicated toolbox to solve the inverse problem.

In this paper we propose an algorithm, called Hybrid Method, initially developed for sound field extrapolation [10], based on an inverse problem with beamforming regularization matrix. The idea of this regularization is to more strongly penalize the non-signal region in the inverse problem. Moreover, there is no assumption on the nature of the sources (correlated or uncorrelated) and the computation time is low if few scan points are required. The aim of this study is to compare the noise source maps provided by the inverse problem regularized with beamforming matrix and other high-resolution source identification algorithms such as Clean-SC and DAMAS. In Section 2, the Hybrid Method is introduced. The performances of the source identification algorithms are compared with synthetic data in Section 3. Section 4 is devoted to the experimental tests and results collected in a closed wind-tunnel for various types of sources.

2. Theory

2.1. Direct problem

We consider S acoustic monopole sources at location $\mathbf{x}_s$ and M microphones at location $\mathbf{x}_m$. The acoustic pressure recorded by the microphones is denoted $\hat{\mathbf{p}}(\mathbf{x}_m)$ and can be written in matrix form as

$$\hat{\mathbf{p}}(\mathbf{x}_m) = \mathbf{G}(\mathbf{x}_m, \mathbf{x}_s)\mathbf{q}(\mathbf{x}_s), \tag{1}$$

where $\mathbf{G}(\mathbf{x}_m, \mathbf{x}_s) = (1/4\pi\|\mathbf{x}_m - \mathbf{x}_s\|)\exp(-\mathrm{j}k\|\mathbf{x}_m - \mathbf{x}_s\|)$ is the free-field Green function representing the acoustic radiation between the sources and the microphones, $\mathbf{q}(\mathbf{x}_s)$ is the strength of the sources and k is the acoustic wavenumber. Bold characters are vectors or matrices and the sizes are $\mathbf{p}(\mathbf{x}_m) \in \mathbb{C}^{M\times 1}$, $\mathbf{G}(\mathbf{x}_m, \mathbf{x}_s) \in \mathbb{C}^{M\times S}$ and $\mathbf{q}(\mathbf{x}_s) \in \mathbb{C}^{S\times 1}$.

2.2. Inverse problem formulation

Basically, the aim of inverse methods is to estimate the strength of the source $\mathbf{q}$ that creates the acoustic pressure $\hat{\mathbf{p}}$ onto the microphone array, knowing the free-field Green function $\mathbf{G}$. One way to define the acoustical inverse problem is to solve a minimization problem that can be written in the general form

$$\mathbf{q}_\lambda = \operatorname{argmin}\{\|\hat{\mathbf{p}} - \mathbf{G}\mathbf{q}\|_2^2 + \lambda^2\Omega(\mathbf{q})^2\}, \tag{2}$$

with $\Omega(\cdot)$ a discrete smoothing norm [11] and λ a regularization parameter. The argument of the minimization problem is a Hermitian quadratic function of $\mathbf{q}$. The vector 2-norm is denoted by $\|\cdot\|_2$

In a similar manner, we introduce a modified version of the Eq. (2) with data scaling by a scalar β

$$\mathbf{q}_{\lambda\beta} = \operatorname{argmin}\{\|\beta\hat{\mathbf{p}} - \mathbf{G}\mathbf{q}\|_2^2 + \lambda^2\Omega(\mathbf{q})^2\}. \tag{3}$$

In Eq. (3), the primary goal of beta β is to compensate for reduced overall source amplitudes caused by over-regularization. This type of scalar compensation was also introduced by Moorhouse [12] with a different approach. The selection of the smoothing norm Ω, the regularization parameter λ and the scaling parameter β are explained in the following sections.

2.3. Inverse problem solution

It is well known that the minimization problem without regularization is ill-conditioned [13–15], meaning that the solution can be very sensitive to measurement noise or model uncertainties, therefore a discrete smoothing norm is used to regularize the unknown solution $\mathbf{q}_{\lambda\beta}$. We can define the discrete smoothing norm as

$$\Omega(\mathbf{q}) = \|\mathbf{L}\mathbf{q}\|_2, \tag{4}$$

where $\mathbf{L}$ is a square regularization matrix. The optimal solution of this minimization problem is obtained by setting the derivative with respect to $\mathbf{q}$ of the cost function equation (3) to zero [16,17]

$$\mathbf{q}_{\lambda\beta} = (\mathbf{G}^H\mathbf{G} + \lambda^2\mathbf{L}^H\mathbf{L})^{-1}\mathbf{G}^H\beta\hat{\mathbf{p}}, \tag{5}$$

where $(\cdot)^H$ is the Hermitian transpose. The solution of the minimization problem can be rewritten as

$$\mathbf{q}_{\lambda\beta} = \mathbf{L}^{-1}([\mathbf{L}^{-1}]^T\mathbf{G}^H\mathbf{G}\mathbf{L}^{-1} + \lambda^2\mathbf{I})^{-1}[\mathbf{L}^{-1}]^T\mathbf{G}^H\beta\hat{\mathbf{p}}, \tag{6}$$

where $(\cdot)^T$ denotes matrix transposition. Therefore the general form inverse problem $\mathbf{q}_{\lambda\beta}$ can be related to the standard form inverse problem $\underline{\mathbf{q}}_{\lambda\beta}$ thanks to the regularization matrix

$$\mathbf{q}_{\lambda\beta} = \mathbf{L}^{-1}\underline{\mathbf{q}}_{\lambda\beta} = \mathbf{L}^{-1}(\underline{\mathbf{G}}^H\underline{\mathbf{G}} + \lambda^2\mathbf{I})^{-1}\underline{\mathbf{G}}^H\beta\hat{\mathbf{p}}, \tag{7}$$

with $\underline{\mathbf{G}} = \mathbf{G}\mathbf{L}^{-1}$. The standard form inverse problem solution $\underline{\mathbf{q}}_{\lambda\beta}$ can be seen as a regularized inverse problem solution whereas the general form inverse problem $\mathbf{q}_{\lambda\beta}$ is regularized and scaled.