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# Modeling and analysis of stick-slip and bit bounce in oil well drillstrings equipped with drag bits

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A B S T R A C T

Rotary drilling systems equipped with drag bits or fixed cutter bits (also called PDC), used for drilling deep boreholes for the production and the exploration of oil and natural gas, often suffer from severe vibrations. These vibrations are detrimental to the bit and the drillstring causing different failures of equipment (e.g., twist-off, abrasive wear of tubulars, bit damage), and inefficiencies in the drilling operation (reduction of the rate of penetration (ROP)). Despite extensive research conducted in the last several decades, there is still a need to develop a consistent model that adequately captures all phenomena related to drillstring vibrations such as nonlinear cutting and friction forces at the bit/rock formation interface, drive system characteristics and coupling between various motions. In this work, a physically consistent nonlinear model for the axial and torsional motions of a rotating drillstring equipped with a drag bit is proposed. A more realistic cutting and contact model is used to represent bit/rock formation interaction at the bit. The dynamics of both drive systems for rotary and translational motions of the drillstring, including the hoisting system are also considered. In this model, the rotational and translational motions of the bit are obtained as a result of the overall dynamic behavior rather than prescribed functions or constants. The dynamic behavior predicted by the proposed model qualitatively agree well with field observations and published theoretical results. The effects of various operational parameters on the dynamic behavior are investigated with the objective of achieving a smooth and efficient drilling. The results show that with proper choice of operational parameters, it may be possible to minimize the effects of stick-slip and bit-bounce and increase the ROP. Therefore, it is expected that the results will help reduce the time spent in drilling process and costs incurred due to severe vibrations and consequent damage to equipment.



## 1. Introduction

A rotary drilling system equipped with a drag bit (fixed cutter bits usually referred to as polycrystalline diamond compact or PDC bits) is one of the bit types used to drill deep boreholes for the production and the exploration of oil and natural gas. Field measurements [1,2] show that the drilling systems with drag bits are more prone to different types of oscillations: namely, lateral, axial and the torsional modes of vibration can be observed with large amplitudes. The main causes of these

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**Nomenclature**

| | |
|---|---|
| $A$ | Cross sectional area of drill line (m$^2$) |
| $b_o$ | The acceleration term in hyperbolic tangent function (-) |
| BHA | Bottom hole assembly (-) |
| $C_{rt}$ | Torsional internal damping in gearbox (N m s) |
| $C_V$ | The effective damping due to fluid motion around the drill string (N m s) |
| $C_{\text{ds}}$ | Axial drill string damping (N s/m) |
| $d$ | Depth of cut (m) |
| $D_o$ | Drill line diameter (m) |
| $E_s$ | Modulus of elasticity of the drill line (Pa) |
| $F_{\text{break}}$ | The friction force applied on draw works drum (N) |
| $F_{\text{hook}}$ | Hook load (N) |
| $f(\dot{x}_a)$ | Heaviside function used to capture the boundary condition of WOB$_f$ that depends on axial velocity in Phase II (-) |
| $f(\dot{\theta})$ | Heaviside function used to capture the TOB friction component sign variations (-) |
| g | Acceleration of gravity (m/s$^2$) |
| $I$ | Draw works motor current (A) |
| $I_m$ | Rotary table motor current (A) |
| $J_{\text{DW}}$ | Inertia of draw works drum (kg m$^2$) |
| $J_{\text{ms}}$ | Inertia of draw works motor shaft (kg m$^2$) |
| $J$ | Lumped inertia of the drill string (kg m$^2$) |
| $J_{\text{rt}}$ | Inertia of the rotary table (kg m$^2$) |
| $J_m$ | Inertia of the rotary table motor shaft (kg m$^2$) |
| $J_{\text{BHA}}$ | Inertia of BHA (kg m$^2$) |
| $k_c$ | Rock linear contact stiffness (N/m) |
| $K_m$ | Draw works motor constant (V s) |
| $K_{\text{mo}}$ | Rotary table motor constant (V s) |
| $K_s$ | The drill line stiffness (N/m) |
| $K$ | The effective torsional stiffness of the drill string (N m/rad) |
| $k_{\text{ds}}$ | Drill string axial stiffness (N/m) |
| $L_p$ | Drill-pipes length (m) |
| $L_b$ | Bottom hole assembly (BHA) length (m) |
| $L_o$ | Initial length of the drill-line from the crown block to the traveling block (m) |
| $L_c$ | Draw works armature inductance (H) |
| $L_i$ | Rotary table armature inductance (H) |
| $L$ | The length of drill line from the crown block till the traveling Block (m) |
| $m_{\text{Top}}$ | Suspension mass (kg) |
| $m_a$ | The drill string effective mass (kg) |
| $m_f$ | Fluid mass (kg) |
| $m_{\text{BHA}}$ | BHA mass (kg) |
| $n$ | Number of blades (-) |
| $n_{\text{DW}}$ | Draw works motor gearbox ratio (-) |
| N | Number of times the drill line runs between the crown block and the traveling block (-) |
| $n_o$ | Rotary table motor gearbox ratio (-) |
| $R_b$ | Bit radius (m) |
| $R_{\text{DW}}$ | Draw works drum radius (m) |
| $R_m$ | Draw works armature resistance (Ω) |
| $R_{\text{mo}}$ | Rotary table armature resistance (Ω) |
| ROP | Rate of penetration (m/s) |
| TOB | Down Hole Torque On Bit (N m) |
| TOB$_f$ | Torque On Bit friction component (N m) |
| TOB$_c$ | Torque On Bit cutting component (N m) |
| $t_n$ | The time required for the bit to rotate by angle of $(2\pi/n)$ (s) |
| $T_{\text{mo}}$ | The torque at the motor shaft driving the rotary table (N m) |
| $T_m$ | The torque at the motor shaft driving the draw works drum (N m) |
| $T_L$ | The torque applied on draw works drum from suspended weight (N m) |
| $T_{\text{DW}}$ | The torque transformed from the motor shaft to the draw works drum (N m) |
| $T_{\text{fr}}$ | The kinematic friction torque applied on draw works drum through arm break (N m) |
| $V_{\text{DL}}$ | The effective drill line velocity (m/s) |
| $V_d$ | Desired suspension mass speed (m/s) |
| $V_{\text{CD}}$ | The supplied rotary table motor voltage |
| $V_c$ | The supplied draw works motor voltage |
| $W_{\text{dd}}$ | Desired draw works drum torsional speed (rad/s) |
| $w_d$ | Rotary table desired speed (rad/s) |
| WOB | Down Hole Weight On Bit (N) |
| WOB$_f$ | Weight On Bit friction component (N) |
| WOB$_c$ | Weight On Bit cutting component (N) |
| $x_a$ | The axial response of the bit (m) |
| $x_{\text{Top}}$ | The axial response of the suspension mass (m) |
| $x_{\text{DL}}$ | The drill line displacement (m) |
| $\gamma$ | Spatial orientation of wear flats (-) |
| $\varepsilon$ | Intrinsic specific energy (Pa) |
| $\xi$ | Inclination of cutting force on the cutting face (-) |
| $\theta_{\text{DW}}$, $\dot{\theta}_{\text{DW}}$ | The angular displacement and velocity of draw works drum (rad, rad/s) |
| $\theta, \dot{\theta}$ | The angular displacement and velocity of the bit (rad, rad/s) |
| $\ell_n$ | Wear flat length beneath each blade (m) |
| $\mu$ | Friction coefficient between rock formation and bit (-) |
| $\mu_P$ | Draw works break pad friction coefficient (-) |
| $\mu_f$ | Viscosity of drilling mud (N s/m$^2$) |
| $\upsilon$ | Poisson's ratio of drilling line (-) |
| $\xi_o$ | Axial damping ratio (-) |
| $\rho_b$ | Drill string material density (kg/m$^3$) |
| $\rho_f$ | Mud density (kg/m$^3$) |
| $\sigma$ | Rock normal contact stress (Pa) |

vibrations include contact and friction at the borehole/drillstring and bit/rock formation interfaces, eccentricity, imbalance, initial curvature in the drill collar sections, and various linear or nonlinear resonances. These severe vibrations often cause failures of drillstrings, abrasive wear of tubulars, damage of the bit, reduction of the rate of penetration (ROP), and consequently incur high costs [3–6]. This paper concentrates on the axial and torsional modes of vibrations, bit bounce and