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Modeling of an air-backed diaphragm in dynamic pressure sensors: Effects of the air cavity

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ABSTRACT

As the key structure of most dynamic pressure sensors, a diaphragm backed by an air cavity plays a critical role in the determination of sensor performance metrics. In this paper, we investigate the influence of air cavity length on the sensitivity and bandwidth. A continuum mechanics model neglecting the air viscous effect is first developed to capture the structural–acoustic coupling between a clamped circular diaphragm and a cylindrical backing air cavity. To facilitate sensor design, close-form approximations are obtained to calculate the static sensitivity and the fundamental natural frequency of the air-backed diaphragm. Parametric studies based on this analytical model show that the air cavity can change both the effective mass and the effective stiffness of the diaphragm. One new finding is that the natural frequency of the air-backed diaphragm behaves differently in three different cavity length ranges. In particular, due to the mass effect of the air cavity being dominant, it is shown for the first time that the natural frequency decreases when the cavity length decreases below a critical value in the short cavity range. Furthermore, a finite element method (FEM) model is developed to validate the continuum mechanics model and to study the damping effect of the air cavity. These results provide important design guidelines for dynamic pressure sensors with air-backed diaphragms.

1. Introduction

Dynamic pressure sensors have been widely used in a variety of consumer, commercial, and military applications including telecommunication [1], speech recognition [2], hearing aids [3], and sound source localization [4]. In terms of transduction methods, these sensors can be based on piezoelectric [5,6], piezoresistive [7], optical [8–10], and capacitive principles [11–13]. For almost all of these sensors, the first stage of transduction involves the deflection of a flexible diaphragm in response to a net differential pressure across its thickness [14,15]. On the backside of the diaphragm, there exists a cavity that is most often filled with air. In general, the air cavity has the following effects that are important to the performance of a pressure sensor: (i) it provides damping to the diaphragm motion due to the viscosity of air (*i.e.*, resistance to the air flow in the cavity), (ii) it increases the effective stiffness of the diaphragm due to the air spring effect, and (iii) it increases the effective mass of the diaphragm due to air particles moving together with the diaphragm.

Because the air cavity plays a critical role in determination of sensor performance, it is imperative to study the mechanics of an air-backed diaphragm, which is the key structure for most dynamic pressure sensors. This is particularly important as

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Nomenclature

a	radius of the diaphragm
A_m	coefficient to normalize $U_d(r)$
$\mathbf{B}_d$	vector used to superpose the modal coordinates of the diaphragm displacement
B_n	coefficients used to normalize $U_a(r)$
c_0	speed of sound in the air
c_d	speed of longitudinal wave in the diaphragm
D	flexural rigidity of the diaphragm
E_d	Young's modulus of the diaphragm
f_{c1}	fundamental natural frequency of the air backed diaphragm
h_d	thickness of the diaphragm
$H_{a,n}$	transfer function relating the displacement $\tilde{w}_a$ to the reaction pressure $\tilde{p}_R$
$H_{c,mn}$	transfer function of the coupled system as defined in Eq. (23)
$\mathbf{H}_c$	transfer function matrix of the coupled system as defined in Eq. (32)
$H_{d,m}$	transfer function of the diaphragm for the mth mode
I_0, $I_0{}'$	modified Bessel function of the first kind and its derivative
$\mathbf{I}_M$	identity matrix of order M
J_0, $J_0{}'$	Bessel function of the first kind and its derivative
$\mathbf{K}_a$	stiffness matrix of the air cavity as defined in Eq. (24)
$\mathbf{K}_c$	stiffness matrix of the coupled system as defined in Eq. (33)
$\mathbf{K}_d$	stiffness matrix of the diaphragm as defined in Eq. (28)
l	length of the air cavity
$(l/a)_{cr,ss}$	critical cavity length when the air cavity and the diaphragm have equal stiffness
$(l/a)_{cr,\mathrm{long}}$	critical cavity length separating the long and medium cavity length regions
$(l/a)_{cr,\mathrm{short}}$	critical cavity length separating the medium and short cavity length regions
m,n	order of mode shape
$M_{a,n}$	equivalent mass of the air cavity as defined in Eq. (27)
$\tilde{M}_{a,n}$	equivalent mass of the air cavity as defined in Eq. (20)
$\mathbf{M}_a$	mass matrix of the air cavity as defined in Eq. (25)
$\mathbf{M}_c$	mass matrix of the coupled system as defined in Eq. (34)
$\mathbf{M}_d$	mass matrix of the diaphragm as defined in Eq. (29)
N_0	in-plane force of the diaphragm
$\mathbf{N}_d$	vector in expanding the applied pressure in terms of the modal coordinates of the diaphragm
p_0	static pressure of the air
p_d	net pressure applied to the diaphragm
$\tilde{p}_d$	net pressure normalized by Young's modulus of the diaphragm
p_e	pressure applied to the external surface of the diaphragm
p_R	reaction pressure at the diaphragm–air interface
$\tilde{p}_R$	reaction pressure normalized by the static pressure p_0
$P_{ed,m}$	coefficients in expanding p_e in terms of the diaphragm's modes
$\tilde{\mathbf{P}}_{ed}$	vector of the normalized pressure applied to the top surface of the diaphragm
$P_{Ra,n}$	modal coefficients in expanding p_R in terms of the air cavity's modes
$P_{Rd,m}$	coefficients in expanding p_R in terms of the air cavity's modes
Q	non-dimensionalized variables as defined in Eq. (49)
Q_1–Q_4	non-dimensionalized variables as defined in Eq. (47)
r	normalized radial coordinate, $0 \le r \le 1$
s_{dyn}	dynamic sensitivity of pressure sensors
t	time
T_{mn}	transformation coefficients between the modes of the diaphragm and the air cavity
$\mathbf{T}$	matrix whose elements are T_{mn}
$U_a(r)$	radial part of the mode shape of the air cavity
$U_d(r)$	radial part of the diaphragm's mode shape
w_a	displacement within the air cavity
$\tilde{w}_a$	displacement w_a normalized by the radius a
w_d	transverse displacement of the diaphragm
$\tilde{w}_d$	transverse diaphragm displacement normalized by the diaphragm radius a
$W_{a,n}$	modal coefficients in expanding w_a in terms of the air cavity's modes
$W_{d,m}$	coefficients in expanding w_d in terms of the diaphragm's modes
$\tilde{\mathbf{W}}_d$	vector of the normalized displacement of the diaphragm
z	normalized axial coordinate, $0 \le z \le 1$
$Z(z)$	axial part of the mode shape of the air cavity
α_1, α_2	variables in the characteristic equation of the diaphragm
β	variable in the characteristic equation for the air cavity
χ	normalized tension parameter of the diaphragm
δ_{mn}	Kronecker delta, $\delta_{mn}=0$ for $m \ne n$; $\delta_{mn}=1$ if $m=n$
γ	adiabatic index of the air
λ	sound wavelength in the air
ν	Poisson's ratio of the diaphragm
θ	Azimuthal coordinate
ρ_0	static density of the air
ρ_d	density of diaphragm
σ	a non-dimensionalized variable as defined in Eq. (45)
ω	radial frequency
ω_d	natural frequency of the diaphragm
ξ	damping ratio
ζ	normalized parameter as defined by Eq. (11)
ϑ	a non-dimensionalized variable as defined in Eq. (39)
Λ	natural frequency parameter of the diaphragm