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Free vibration of three-dimensional multilayered magneto-electro-elastic plates under combined clamped/free boundary conditions

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ABSTRACT

In this paper, we study the free vibration of multilayered magneto-electro-elastic plates under combined clamped/free lateral boundary conditions using a semi-analytical discrete-layer approach. More specifically, we use piecewise continuous approximations for the field variables in the thickness direction and continuous polynomial approximations for those within the plane of the plate. Group theory is further used to isolate the nature of the vibrational modes to reduce the computational cost. As numerical examples, two cases of the lateral boundary conditions combined with the clamped and free edges are considered. The non-dimensional frequencies and mode shapes of elastic displacements, electric and magnetic potentials are presented. Our numerical results clearly illustrate the effect of the stacking sequences and magneto-electric coupling on the frequencies and mode shapes of the anisotropic magneto-electro-elastic plate, and should be useful in future vibration study and design of multilayered magneto-electro-elastic plates.

1. Introduction

Multilayered composites offer many useful features as structural components and as such, response of such composites under external loads is an important research subject. Besides the common numerical methods, such as the finite-element and boundary-element methods, various analytical/semi-analytical solutions were presented for layered composite plates. For instance, Vel and Batra [1] presented an analytical three-dimensional (3D) solution for the static deformation of multilayered piezoelectric plates under general boundary conditions in terms of series expansion. The corresponding bending vibration was further solved by Vel et al. [2]. The extended Kantorovich method was also applied to the static bending of layered piezoelectric plates by Kapuria and Kumari [3], and to the 3D deformation of layered elastic plates by Kumari et al. [4] where an iterative scheme was employed.

Recent development of smart materials/structures is receiving widespread attention owing to their potential applications in various engineering fields such as sensors, actuators and microwave devices. As an important member of these smart materials, magneto-electro-elastic (MEE) materials which consist of piezoelectric (PE) and piezomagnetic (PM) phases, are able to facilitate the energy conversion between the electric and magnetic fields. Such a phenomenon is called

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magnetoelectric (ME) effect which cannot be found in the pure piezoelectric or piezomagnetic material. Since the report on the ME effect by van Suchtelen [5], many interesting studies on MEE materials and structures have been carried out. Among them, the static and dynamic behaviors of typical MEE structures, for example plates and beams, are especially investigated.

For a simply supported multilayered MEE plate, the exact closed-form solution of the deformation under a static mechanical load was derived using the pseudo Stroh formalism [6]. The corresponding free vibration was analyzed by Pan and Heyliger [7]. Another method, the state-space formulation, was also widely used in the analysis of the static and dynamic behaviors of MEE multilayered plates [8,9]. Free vibration of a non-homogeneous transversely isotropic MEE plate was carried out by Chen et al. [10]. Besides, the discrete-layer and domain-discretization methods were also proposed to the analysis of free vibration of anisotropic elastic and MEE plates and shells [11–15].

For a simply supported MEE plate, analytical solutions of the field variables can be found that satisfy exactly the lateral boundary conditions. However, under other lateral boundary conditions such as the clamped or free conditions, one cannot find such analytical expressions of the field variables. Furthermore, many commercial finite-element codes cannot handle the multiphase coupling problem. Thus, in this paper, a semi-analytical discrete-layer model of the governing differential equations is developed and applied to typical layered MEE media under combined clamped and free lateral boundary conditions. Our representative numerical results on the natural frequencies and mode shapes clearly show the unique characteristics of these MEE solids which should be of particular interest to the design of layered MEE composites.

2. Formulation

2.1. Governing equations

While our semi-analytical model can be applied to any layered plate, we consider an anisotropic, MEE, and three-layered rectangular plate with horizontal dimensions a and b and total thickness H (in the vertical or thickness direction) as shown in Fig. 1. A Cartesian coordinate system is attached to the plate and its origin is at one of the four corners on the bottom surface, with the plate occupying the region of $z \geq 0$. The interface of each layer is assumed to be bonded perfectly. In other words, the elastic displacements, electric and magnetic potentials, elastic traction, and the z-components of the electric displacement and magnetic induction are continuous across the interfaces.

For a linear, anisotropic MEE solid, the coupled constitutive equation can be written in the following form:

$$\sigma_i = c_{ik}\gamma_k - e_{ki}E_k - q_{ki}H_k,\; D_i = e_{ik}\gamma_k + \varepsilon_{ik}E_k + d_{ik}H_k,\; B_i = q_{ik}\gamma_k + d_{ik}E_k + \mu_{ik}H_k, \tag{1}$$

where σ_i, D_i and B_i are the stress, electric displacement and magnetic induction, respectively; γ_k, E_k and H_k are the strain, electric field and magnetic field, respectively; c_{ik}, ε_{ik} and μ_{ik} are the elastic, dielectric, and magnetic permeability coefficients, respectively; e_{ik}, q_{ik} and d_{ik} are the piezoelectric, piezomagnetic and magnetoelectric coefficients, respectively. We remark that various uncoupled cases can be reduced by setting the appropriate coupling coefficients to zero.

The relationship between the strain and displacement, electric (magnetic) field and its potential can be expressed as

$$\gamma_{ij} = 0.5(u_{i,j} + u_{j,i}),\; E_i = -\varphi_i,\; H_i = -\psi_{,i}, \tag{2}$$

where u_i are the elastic displacements, and φ and ψ are the electric and magnetic potentials, respectively. The subscript after the comma, e.g. ",i", in the elastic displacement, electric and magnetic potentials denotes partial derivative with respect to the i-th coordinate x_i ($x_1 = x$, $x_2 = y$, $x_3 = z$).

For the problem to be considered in this paper, we assume that the body forces, electric charge and current densities are zero; thus the governing equations of motion in the dynamic case are given by

$$\sigma_{ij,j} = \rho\frac{\partial^2 u_i}{\partial t^2},\; D_{j,j} = 0,\; B_{j,j} = 0, \tag{3}$$

where ρ is the density of the material.

While general conditions may be prescribed to the lateral boundaries (along the whole thickness of the plate; i.e., for any given z-coordinate in the problem domain) of the layered plate, we consider the following two typical cases.

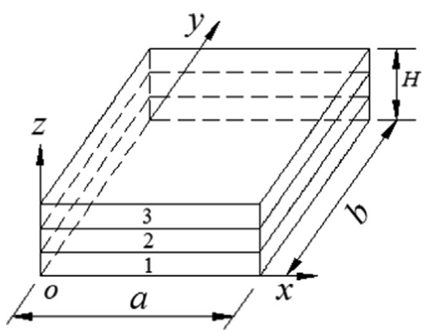

Fig. 1. The three-layered magneto-electro-elastic plate.