Journal of Sound and Vibration 333 (2014) 5412–5426

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Damage localisation in plate like-structures using the two-dimensional polynomial annihilation edge detection method

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ARTICLE INFO

Article history:
Received 12 November 2013
Received in revised form
8 May 2014
Accepted 22 May 2014
Handling Editor: I. Trendafilova
Available online 27 June 2014

ABSTRACT

The topic of non-destructively detecting localised damage in plates is addressed in this article. Since the presence of a crack or a delamination causes a discontinuity in the mode shape first derivatives, a numerical method for detecting discontinuities in smooth piecewise functions and their derivatives, based on a polynomial-annihilation technique is presented. The method, already proposed for beam-type structures, has been extended to enable the detection and localisation of damage in plate-like structures for which only post-damage mode shapes are available. Applying finite element analysis, the mode shapes of an isotropic plate with a saw-cut and a multi-layered composite plate with a delamination have been calculated and the performance of the approach evaluated for increasing amounts of noise. Encouraging results indicate that further development of the technique for non-destructive testing of plate-like structures would be highly worthwhile.

1. Introduction

In quite recent years there has been a significant effort to employ vibration-based inspection methods for identifying various types of structural damage. Since the occurrence of damage modifies the stiffness, the mass and the damping properties of a structure, its vibrational response will also change as a result of damage occurring. By appropriately assessing the eigen-parameters of a damaged structure, it may be possible to detect, locate, and even quantify the extent of damage. One class of damage identification techniques is based on the modifications of mode shapes due to damage or, alternatively, on the changes of the mode shape second derivatives, i.e., curvatures. These methods appear to be more sensitive than methods based on the natural frequency changes [1].

Most of the techniques have been mainly applied to beam-like structures but more recently the attention has been focused also on two-dimensional structures such as plates or shells.

Cornwell et al. [2] extended the technique developed by Stubbs et al. [3] based on the decrease in modal strain energy as defined by the curvature of the measured mode, to isotropic plates. One of the drawbacks of this method is that data from the undamaged structure are required for the necessary comparison with the data of the damaged structure.

Ratcliffe [4] proposed the so-called 'Gapped-Smoothing Technique' that enables damage detection in beam structures by evaluating a damage index which is a function only of the modal curvature calculated for the damaged state. The advantage is that no a-priori knowledge regarding the undamaged state is required. Battipede et al. [5], simultaneously with

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http://dx.doi.org/10.1016/j.jsv.2014.05.047
0022-460X/

Yoon et al. [6], extended the 'Gapped-Smoothing Technique' to plate-like structures, and the latest developments were published in [7].

Evolving the idea of an interpolation technique that can smooth the discontinuities in the data due to delamination, Smoothing Element Analysis (SEA) has been used and a damage index, that considers the change in strain energy due to the delamination in the multilayered panel, has been formulated [8].

In general, methods based on the mode shape second derivatives seem to demonstrate a higher level of sensitivity to damage than those based on mode shapes themselves. Unfortunately, the weakness of these methods is that, being based on a double differentiation procedure, they are highly sensitive also to the presence of measurement noise. Although the displacement mode shape is generally not sufficiently sensitive to small damage [9], recently new methods based on spatial wavelet analysis have emerged as a potential way of overcoming this problem thanks to their high-resolution. Indeed, although local changes in the mode shapes due to cracks or defects are not easily detectable, it may be possible to identify singularities in the signal by applying the wavelet transform to the mode shapes. This procedure has been applied to both one- [10–12] and two-dimensional structures [13]. Latterly a method based on Gaussian Process Regression has been proposed by Hensman et al. [14] and applied to the displacement mode shape of a beam with a crack to identify damage.

In [15] Surace presents a review of past and present research studies in which the author has been involved that aim to detect the presence of structural damage and identify its approximate location, using only post-damage mode shapes and their derivatives in the cases of uni- and bi-dimensional structures.

In this paper, a new methodology to localise damage in plate-like structures, for which only few post-damage mode shapes are available, is proposed. It is well known that a kind of damage such as a crack or a delamination, causes a discontinuity in the rotations and consequently in the mode shape first derivatives. On this basis, the Polynomial Annihilation Edge Detection (PAED) technique, a numerical method for detecting discontinuities in smooth piecewise functions [16] and their derivatives [17,18], already applied by Surace et al. [19–21] to the problem of damage detection and localisation in beam-like structures, has been extended to bi-dimensional case (PAED2). Two case studies have been considered. In the first case, an isotropic clamped plate with a saw-cut has been studied. In correspondence with the saw-cut, the mode shape first derivatives present a jump discontinuity which can be detected using the PAED2. In the second case, a multilayered composite panel presenting a delamination has been analysed. Due to this kind of damage, two sub-laminates are created respectively. Consequently, especially at high frequencies, the local modes exhibit discontinuities in the rotation function where the sub-laminates are separated. For both case studies the dynamic behaviour of the plates has been simulated using a finite element model and some results are given for increasing amount of noise.

2. Polynomial annihilation edge detection method for one-dimension

A crack or a localised damage in a structure provokes a discontinuity in the rotation (Fig. 1(a)). Consequently, the mode shape is not-smooth at the damage position (Fig. 1(b)) and the first derivative (strictly related to the rotation) presents a jump discontinuity in correspondence with the damage (Fig. 1(c)). Based on this simple concept, the polynomial annihilation edge derivative detection method for one dimension was proposed in [17] (as an extension of the Polynomial Annihilation Edge Detection – PAED [16]) in order to predict the locations of the derivative discontinuities. The technique was then applied to the problem of damage localisation in beam-like structures in [19–21] where the detailed theory for the one-dimensional case is presented. Here for the sake of clarity, the main concepts of the method are reported.

Essentially, given a piecewise smooth function $f(x)$ defined on the interval $[a,b]$ and known only on the set of discrete points, $S=\{x_1,x_2,...,x_N\}\subset(a,b)$, it is assumed that for $\gamma\in\{1,2,...\}$ γ-derivation jump discontinuity is considered in $f(x)$, i.e., all its derivatives up to $f^{(\gamma-1)}(x)$ are continuous in $[a,b]$ and a jump discontinuity first appears in $f^{(\gamma)}(x)$ for $x=\xi$. It is also assumed that at any point x in the domain, $f^{(\gamma)}(x)$ has well-defined one sided limits. The *local jump function* for the γ^{th} derivative is defined as $[f^{(\gamma)}](x)=f^{(\gamma)}(x+)-f^{(\gamma)}(x-)$. This function is different from 0 only in correspondence with the discontinuity where $f^{(1)}(\xi+)\neq f^{(1)}(\xi-)$. The method, using the polynomial annihilation property [16–18], allows to find an approximation of the local jump function $[f^{(\gamma)}](x)$ that converges rapidly to zero away from the discontinuity only with the knowledge of $f(x_1)$, …, $f(x_N)$. In the case of the damage location procedure applied to beam-like structures, $f(x)$ is a mode shape of the damage beam, where x is the coordinate along its longitudinal axis and the discontinuity concerns the first derivative, i.e. $\gamma=1$, as it is shown in Fig. 1.

3. Polynomial annihilation edge detection method for two-dimensions (PAED2)

In a two-dimensional case, which is of relevance to the current paper, the method can be used for derivative edge detection if the data are given on a structured grid. A brief description of this implementation is presented for the first time here.

Let $f(x^{(1)},x^{(2)})$: $\Omega=[a,b]^2\rightarrow\mathbb{R}^2$ be a piecewise smooth function defined on a bounded domain $\Omega\subseteq\mathbb{R}^2$ known only on a set of discrete points $S=\{\mathbf{x}_1,\dots,\mathbf{x}_N\}\subseteq\Omega$. For ease of presentation, it is assumed that $f\in C^{(\gamma-1)}$, i.e. $f(\mathbf{x})$ and all its derivatives up to $f^{(\gamma-1)}$ are continuous in Ω. Of interest is the γ- derivative discontinuity detection in $f(x^{(1)},x^{(2)})$, $\gamma\in N=\{1,2,\dots\}$. It is also supposed that at any point $\mathbf{x}\in\Omega$, f has well defined one-sided limits.