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On the nonlinear primary resonances of a piezoelectric laminated micro system under electrostatic control voltage

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ABSTRACT

In this article, a comprehensive nonlinear analysis for a piezoelectric laminated micro system around its static deflection is presented. This static deflection is created by an electrostatic DC control voltage through an electrode plate. The micro system beam is assumed as an elastic Euler-Bernoulli beam with clamped-free end conditions. The dynamic equations of this model have been derived by using the Hamilton method and considering the nonlinear inertia, curvature, piezoelectric and electrostatic terms. The static and dynamic solutions have been achieved by using the Galerkin method and the multiple-scales perturbation approach, respectively. The results are compared with numerical and other existing experimental results. By studying the primary resonance excitation, the effects of different parameters such as geometry, material and excitations voltage on the system's softening and hardening behaviors are evaluated. In a piezoelectrically actuated micro system it was showed that because of existence of curvature and inertia nonlinear terms a small change in excitation amplitude can lead to the formation and expansion of nonlinear response. In this paper, it is demonstrated that by applying an electrostatic DC control voltage, these nonlinearities can be controlled and altered to a linear domain. This model can be used to design a nano or micro-scale smart device.

1. Introduction

The micro-systems, which can be fabricated due to recent advances in integrated circuits, include moving parts such as beams, plates, membranes or other mechanical components. Because of their small size, the easy creation of relatively large amplitude oscillations and the existence of strong effects by nonlinear sources, these components exhibit nonlinear behaviors. The emergence of phenomena such as softening and hardening, hysteresis, jumps in resonance frequency, the existence of more than one stable state, sensitivity to initial conditions and dynamic pull-in in these devices are all cases that cannot be handled by linear theories. Therefore, nonlinear models and theories should be employed to obtain precise

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Nomenclature

a	amplitude of system response under static condition
a_0	amplitude of system response under static condition at equilibrium state
$a[i]$ $i=1\ldots M$	the coefficients obtained for the comparison functions in calculating v_s
A, $\bar{A}$	mixed function, conjugate of A
A_b, A_p	cross section area of the micro-beam and the piezoelectric layer, respectively
$b[j]$ $j=1\ldots M$	the coefficients obtained for the comparison functions in calculating φ
$C_\gamma(s)$	the magnitude of the bending moment due to the piezoelectric effects
$C_\eta(s)$	bending stiffness of the system
d	distance between the micro-beam and the electrode plate
e	stretching strain of the neutral axis bending
E_b, E_p	micro-beam and piezoelectric modulus of elasticity, respectively
$H(x)$	a dimensionless measure for the bending stiffness of the system
H_{lp}	heaviside function at the point l_p
I_b, I_p	the second moment of area for the micro-beam and the piezoelectric layer cross section about the neutral axis, respectively, where $0<s<l_p$,
K	summation of nonlinear terms
l	micro-beam length
l_p	distance to the left end of the piezoelectric layer from the left end of the micro-beam
$m(x)$	a dimensionless measure for the mass per unit length of the micro-beam
$N_e(x)$	effective nonlinear coefficient
p^*, q^*	real and imaginary part of A
q	dimensionless bookkeeping parameter
Q	electrostatic load per unit length of the micro-beam
syz, $\bar{s}\,\bar{x}\,\bar{z}$	fixed and local coordinate systems, respectively
t	time
t_b, t_p	Micro-beam and piezoelectric layer thickness, respectively
T	the time scale which is used to change the time into dimensionless form
T_k	kinetic energy
u	micro-beam longitudinal displacement
v	micro-beam bending displacement
v_d	the added dynamic deflection to v_s
v_s	dimensionless static deflection
$v_s[i]$	ith comparison function used for obtaining v_s
V	potential energy
V_{dc}	applied voltage between the micro-beam and the electrode plate
w_b	width of micro-beam and piezoelectric layer
x	the dimensionless form of s
$\bar{z}$	distance from the neutral axis of cross section
z_n	distance between the neutral axis and the midline of the micro-beam
$\alpha_2 P_{ac}\cos(\Omega t)$	dimensionless measures for piezoelectric actuation
$\alpha_3 V_e^2$	dimensionless measures for electrostatic actuation
γ	phase of system response under static condition at equilibrium state
γ_0	phase of system response under static condition
ε	strain in the cross section of the system
ε_0	dielectric constant between micro-system and fixed electrode
θ	rotation angle between fixed and local coordinates
κ	curvature bending of the micro-beam in the sz-plane
ρ_b, ρ_p	Micro-beam and piezoelectric layer specific density, respectively
σ	detuning parameter
σ_b, σ_p	micro-beam and piezoelectric layer axial stress, respectively
τ	the dimensionless form of t
$\varphi_a[j]$	jth comparison function used for obtaining φ
$\varphi(x)$	mode shape of the system
ω	natural frequency of system

and correct results in these cases. In common micro-systems that include micro-switches and micro-resonators, the main mechanical component is a micro-beam. By the excitation of a DC voltage, a fixed static deformation is formed in this micro-beam and then by using an AC harmonic voltage, the micro-beam's vibration modes are excited, and the micro-beam starts to oscillate at its natural frequency. These voltage excitations can be applied electrostatically or piezoelectrically. In electrostatic excitation with a sufficiently high voltage, pull-in phenomenon can occur, in which case, the micro-beam becomes unstable and sticks to the electrode plate in front of it [1]. The effect of nonlinear terms in governing equations of an electrostatically excited micro-cantilever is presented in many studies. Researchers in their work demonstrated that the effect of nonlinear terms due to inertia and geometry appears as hardening and the effect of electrostatic forces appear as softening on vibration behavior of a micro-system [2,3]. Due to their low weight, fast response, low energy consumption and high bandwidth performance, piezoelectric materials have been extensively used in microsystems as actuators and sensors [4]. Li et al. and Dick et al. in separate studies showed that a piezoelectrically actuated micro system demonstrated a hardening behavior. They use multiple scale perturbation method to solve the dynamic equations [5,6]. By using the Lumped parametric model, Lee et al. studied the mechanical behavior of a micro-cantilever subjected to piezoelectric excitation. They obtained their numerical results by solving the nonlinear Duffing's equation with one degree of freedom and compared them with experimental findings. They concluded that the resonance frequency displays hysteretic behavior as the DC voltage changes,