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Modal interaction activations and nonlinear dynamic response of shallow arch with both ends vertically elastically constrained for two-to-one internal resonance

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ABSTRACT

This study investigates the two-to-one internal resonance of the shallow arch with both ends elastically constraining, and the primary resonance case is considered. The full-basis Galerkin method and the multi-scale method are applied to obtain the modulation equations. It is shown that the natural frequencies of the first two modes cross/avoid to each other when the stiffness of elastic supports at two ends is the same/different. Moreover, the nonlinear modal interactions between these two modes may not/may be activated. The force/frequency-response curves are employed to explore the nonlinear response of the elastically supported shallow arch. The saddle-node bifurcation points and Hopf bifurcation points are observed in these cases. Moreover, the dynamic solutions, i.e., the periodic solution, quasi-periodic solution and chaotic solution are discussed. The numerical simulations are used to illustrate the route to chaos via period-doubling bifurcation.

1. Introduction

Shallow arches are widely used in many engineering fields, i.e., civil engineering, mechanical engineering and aerospace engineering. The large amplitude vibrations may occur when they are designed to low-damping, light-weight and full-load. Therefore, the nonlinear dynamic responses and bifurcations of shallow arches have attracted the interest of many researchers [1–5].

Overall, due to the initial static configuration of the shallow arch, the internal resonance may be activated. In this case, the energy transfer between the involved resonant modes may occur, resulting in complex dynamic behavior. Tien et al. [6,7] studied the bifurcations and chaos of a planar hinged-hinged shallow arch under a harmonic excitation with 1:2/1:1 internal resonances between the lowest two modes by using a 2DOF model. Malhotra and Namachchivaya [8,9] investigated the global dynamics of a planar hinged-hinged shallow arch subjected to a spatially and temporally varying force under principal subharmonic resonance and 1:2/1:1 internal resonance near single mode periodic motions. Moreover, the 1:2 internal resonance of a shallow arch under a periodic excitation is studied by Bi and Dai [10]. Recently, Zulli et al. [11] analyzed the dynamics of curved beams undergoing large oscillations via varying the initial curvature. On the other hand, Thomsen [12] studied the chaotic motions of the two-to-one internal resonance. Benedettini et al. [13] investigated the

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Nomenclature

a_m, β_m	real modulus and phase for A_m
a_n, β_n	real modulus and phase for A_n
A	area of cross section
A_m, A_n, A_k	modulus of mth-, nth-, kth-order mode in first-order approximate solution
b	dimensionless arch rise
c_i	coefficients of mode
cc	conjugate for preceding complex terms
D_i	partial differential to time scales
E	elastic modulus
$\hat{f}$	dimensional vertical periodic load
f_k	excitation amplitude of kth-order mode
F	amplitude of periodic excitation
G_2	quadratic nonlinear operator
G_3	cubic nonlinear operator
i	imaginary unit
I	inertia moment of cross section
$\hat{k}_1, \hat{k}_2$	stiffness of dimensional vertical supports at two ends
k_1, k_2	stiffness of dimensionless vertical supports at two ends
$\hat{l}$	dimensional arch span
$\mathcal{L}$	linear operator
m	lower order of 2:1 internal resonance
n	higher order of 2:1 internal resonance
NST	non-resonant terms
$\hat{o}$	origin of Cartesian coordinate
p_m, q_m	Cartesian form coordinates for A_m
p_n, q_n	Cartesian form coordinates for A_n
r_k	generalized coordinates
R	gyration radius of cross section
S_1, S_2	first-order quadratic coefficients
S_{mm}, S_{nn}, S_{mn}	second-order quadratic coefficients
t	dimensionless time
$\hat{t}$	dimensional time
T_0, T_1, T_2	time scales
u	dimensionless vertical displacements of an arbitrary point
$\hat{u}$	dimensional vertical displacements of an arbitrary point
x, y	x, y-axis of dimensionless Cartesian coordinate
$\hat{x}, \hat{y}$	$\hat{x}, \hat{y}$−axis of dimensional Cartesian coordinate
χ_m, χ_n	detuning terms in Cartesian form modulation equations
δ	Kronecker delta
ε	small parameter
$\gamma_1, \gamma_2, \gamma_3$	phases in polar form modulation equations
$\Gamma_1 \sim \Gamma_5$	coefficients in mode
$\kappa_1 \sim \kappa_6$	shape functions
$\Lambda_{kij}, \Gamma_{kijh}$	coefficients related to modes in Galerkin integration
μ	dimensionless damping parameter
μ_k	damping coefficient of kth-order mode
$\hat{\mu}$	dimensional damping parameter
ω	frequency
ω_k	kth-order frequency
Ω	frequency of periodic excitation
ϕ_k	kth-order mode
π	PI
ψ	dimensionless initial arch axis
$\hat{\psi}$	dimensional initial arch axis
ρ	density
σ_1	detuning parameter between ω_m and ω_n
σ_2	detuning parameter between Ω and ω_m (ω_n)
$\sum$	summation symbol

nonlinear coupling and instability of a non-shallow arch excited by a sinusoidally varying force under 2:1 internal resonance.

Generally speaking, these studies only consider the fixed boundary conditions. However, the real boundary conditions are much more complex than this case in the engineering field. On the other hand, these conditions directly affect the mechanical properties of the arch. Therefore, it is reasonable to treat these conditions as elastic constraint/support boundaries. In this respect, many studies have investigated the instability phenomenon of the snap-through [14,15]. Chen and his coworkers [16,17] studied the dynamic snap of the shallow arch with one end pinned while another end supported by a horizontal spring. Pi and his coworkers [18–20] investigated a series of studies on the planar nonlinear stability of pin-ended shallow arches with various elastic supports. Lee et al. [21] examined the planar free vibration of the rotating curved beam with one end elastically restrained. Moreover, the nonlinear stability of the shallow parabolic arch with horizontal spring supports subjected to the uniform load was investigated [22,23].

Most of these studies focused on the nonlinear stabilities of the shallow arch with complex boundary conditions. Whereas, no study consider the internally resonant response of the shallow arch. In fact, depending on the difficult boundary condition and the initial configuration, the natural frequencies of the shallow arch may exist the integer ratio, i.e., 1:1, 2:1 and 3:1. More importantly, for the shallow arch with elastic supports, if the stiffness of two ends is different, the natural modes do not exhibit any symmetric character [24]. In this case, the first-order coefficient will be different from zero [25,26]. Therefore, the internal resonance of the shallow arch can be activated.

In this study, the two-to-one resonant response of the shallow arch with two ends vertically elastically constrained ends is investigated. The paper is organized as follows. In Section 2 the modulation equations of the shallow arch with elastic supports are obtained, and the eigenvalue analysis on the linear problem is performed. The full-basis Galerkin discretization and multi-scale perturbation analysis are shown in Section 3. Section 4 discusses the modal interaction activation, the equilibrium and dynamic solutions of the modulation equations. Finally, a short summary of results is presented in Section 5.