Journal of Sound and Vibration 333 (2014) 5548–5565

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Reflection of an acoustic line source by an impedance surface with uniform flow ☆

E.J. Brambley [a,*], G. Gabard [b]

[a] Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Wilberforce Road, Cambridge CB3 0WA, United Kingdom
[b] Institute of Sound and Vibration Research, University of Southampton, Highfield, Southampton SO17 1BJ, United Kingdom

ARTICLE INFO

Article history:
Received 7 January 2014
Received in revised form
6 May 2014
Accepted 14 May 2014
Handling Editor: Y. Auregan
Available online 1 July 2014

ABSTRACT

An exact analytic solution is derived for the 2D acoustic pressure field generated by a time-harmonic line mass source located above an impedance surface with uniform grazing flow. Closed-form asymptotic solutions in the far field are also provided. The analysis is valid for both locally-reacting and nonlocally-reacting impedances, as is demonstrated by analyzing a nonlocally reacting effective impedance representing the presence of a thin boundary layer over the surface. The analytic solution may be written in a form suggesting a generalization of the method of images to account for the impedance surface. The line source is found to excite surface waves on the impedance surface, some of which may be leaky waves which contradict the assumption of decay away from the surface predicted in previous analyses of surface waves with flow. The surface waves may be treated either (correctly) as unstable waves or (artificially) as stable waves, enabling comparison with previous numerical or mathematical studies which make either of these assumptions.

The computer code for evaluating the analytic solution and far-field asymptotics is provided in the supplementary material. It is hoped this work will provide a useful benchmark solution for validating 2D numerical acoustic codes.

1. Introduction

Impedance surfaces are an important concept in acoustics, as they can be used to represent the response of any surface to small amplitude perturbations (such as sound or instabilities), and have been successfully employed in applications ranging from outdoor acoustics over a porous ground [2] to acoustic linings in aircraft engine intakes [3]. The mean flow present in aircraft engine intakes complicates the interaction between the acoustics and the impedance surface, potentially allowing for instabilities to grow over the surface [4]. In order to investigate the behavior of sound interacting with an impedance surface, either plane waves or localized (point or line) sources are usually employed. While plane waves are conceptually simpler, localized sources have two important advantages. Firstly, the solution to a point or line source forcing represents

☆ A preliminary version of some parts of this paper was presented as part of AIAA Paper 2013-2218 at the 19th AIAA/CEAS Aeroacoustics Conference in Berlin, Germany [1].

* Corresponding author. Tel.: +44 1223 760457; fax: +44 1223 765900.
E-mail addresses: E.J.Brambley@damtp.cam.ac.uk (E.J. Brambley), G.Gabard@soton.ac.uk (G. Gabard).

http://dx.doi.org/10.1016/j.jsv.2014.05.026
0022-460X/

Green's function and can therefore be used to construct the response of an impedance surface to arbitrary forcing (including, but not limited to, localized dipole and quadrupole forcing). Secondly, localized sources are far better suited to verifying the correctness of computational acoustic simulations, since they involve no incoming waves from outside the computational domain and since they excite all supported acoustic modes (including possibly unstable surface modes). For these reasons, this paper presents an analytic solution to the simplest possible situation involving a localized source, an impedance surface and a mean flow, this being the 2D situation of a uniform flow over an infinite constant-impedance surface subjected to a harmonic line forcing.

There have been many studies of the behavior of an infinite flat impedance surface subjected to a point source in 3D [5–10] and subjected to a line source in 2D [11–14]. None of these studies consider a mean flow. Moreover, all of these studies start with a statement that the solution may be written as a sum of a free-field, or direct, wave, and a reflected wave. Integral identities are then manipulated to find a correction to either a hard-walled ($Z=\infty$) or pressure-release ($Z=0$) reflected wave, or to find the functional form of a reflection coefficient, often involving the manipulation of integrals of Bessel identities and/or the decomposition of cylindrical waves into an integral over plane waves. Here, we use a much simpler and more direct formulation by solving directly the Helmholtz equation using a Fourier transform. The solution derived in this way is naturally found to separate into a direct and a reflected term, and for locally-reacting surfaces the reflected term is naturally of the form of a pressure-release reflection plus a correction.

The classical method of images may be used to compute the reflected wave for a hard-wall or a zero-pressure surface. Morse and Bolt [5, pp. 141–143] found a comparable result for a general impedance surface for a 3D point source, which consisted of an image point source plus an image line source perpendicular to the surface, thus providing a generalization of the method of images. This result has been repeatedly rediscovered, as pointed out by Taraldsen [10], who argued that this image line source is better viewed as a complex line source extending in the imaginary normal direction to the surface. Here, we find a comparable generalization for the reflection of a 2D line source; this generalization of the method of images in 2D is different from the 3D generalization, albeit with certain similarities.

An impedance surface often supports waves itself, which are here referred to as surface waves. For a point source in 3D, Attenborough et al. [7], building on the work of Paul [6], performed a steepest descent analysis and found a pole that gave rise to a surface wave. For a 2D line source, Mechel [14] also found a pole that leads to a surface wave. Here, due to the inclusion of mean flow, the surface wave behavior is expected to be rather different [4,15], with one such surface wave potentially corresponding to a hydrodynamic instability of flow over the surface. Due to this complication, Rienstra and Tester [16] choose to neglect the possible instability of the surface waves as an explicit modeling assumption while deriving their Green's function for a 3D point source in a lined cylindrical duct with flow. Studies of surface waves with flow [4,15,17] have so far investigated only the propagation of surface waves, and have not consider their generation by an explicit acoustic source. Because of this, these studies assumed the surface waves to be localized to the surface and to decay exponentially away from it, which Crighton [18] showed need not be true if the acoustic source is included in the model. Crighton demonstrated this by showing that flow over an elastic plate subjected to a line source results in “leaky waves” that initially grow exponentially away from the surface before eventually being cutoff by a Fresnel region. Here, a similar effect is found.

In addition to providing insight into the nature of the solution, the analysis of a line source above an impedance surface with flow represents a useful benchmark problem against which to validate numerical simulations. Choosing a problem to benchmark against can be difficult. For a realistic situation involving complicated geometries where no exact analytic solution exists, such as a 3D mode propagating out of a realistic aeroengine intake [19], comparisons can only be made against approximate analytic solutions, such as those using the Method of Multiple Scales [20–24]. In such situations, it is unclear how well the analytic and numerical solutions should be expected to match, since the analytic solution is only approximate. Similarly, validating numerical simulations against experimental results, as performed by Richter [19] using the NASA Grazing Incidence Tube experimental results [25], has the disadvantage that the mode excited, the definition of the liner impedance and the measured results are all subject to experimental error, and that moreover the results appear sensitive to other factors such as the tube's downstream exit impedance, 3D effects, and viscous and boundary layer effects [26–28]. Simple test cases therefore allow an exact comparison between the numerical simulation and the analytic solution, one of the simplest of which is that of an oblique plane wave reflecting from an impedance surface [29]; a number of common validations for 2D and 3D time-dependent numerical acoustics simulations with flow are described by Richter [19, chapter 7.1], and similar validations are also used in 2D and 3D frequency-domain simulations [e.g. 30]. However, while these situations are sufficiently simple to permit an exact closed-form analytical expression for the pressure field, all of these tests rely on accurately introducing an incoming acoustic wave at the boundary of the numerical domain, while many of them also do not test the correct simulation of surface waves, do not test the impedance surface at a range of axial wavelengths, nor test correct propagation to the far field. Here, we consider a simple 2D benchmark problem which avoids all of these limitations, yet which still permit an analytic solution. Apart from the implementation of the impedance boundary condition, this benchmark problem is relatively straightforward to simulate numerically, and does not require the forcing of an incoming wave or mode at a numerical boundary.

In Section 2, both exact analytical solutions and leading-order asymptotic far-field solutions are derived directly from the governing equations. The results are valid for both locally- and nonlocally-reacting impedance surfaces, and this is demonstrated in Section 3 by implementing a modified impedance boundary condition [31] which accounts for a thin boundary layer over the impedance surface and results in an effective impedance which is nonlocal. Plots of the results of both of these sections are given in Section 4, before the implications of the analysis for our understanding of surface waves