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On the effect of viscosity and thermal conductivity on sound propagation in ducts: A re-visit to the classical theory with extensions for higher order modes and presence of mean flow

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ABSTRACT

The dispersion equation for the axisymmetric modes of viscothermal acoustic wave propagation in uniform hard-walled circular ducts containing a quiescent perfect gas is classical. This has been extended to cover the non-axisymmetric modes and real fluids in contemporary studies. The fundamental axisymmetric mode has been the subject of a large number of studies proposing approximate solutions and the characteristics of the propagation constants for narrow and wide ducts with or without mean flow is well understood. In contrast, there are only few publications on the higher order modes and the current knowledge about their propagation characteristics is rather poor. On the other hand, there is a void of papers in the literature on the effect of the mean flow on the quiescent modes of propagation. The present paper aims to contribute to the filling of these gaps to some extent. The classical theory is re-considered with a view to cover all modes of acoustic propagation in circular ducts carrying a real fluid moving axially with a uniform subsonic velocity. The analysis reveals a new branch of propagation constants for the axisymmetric modes, which appears to have escaped attention hitherto. The solution of the governing wave equation is expressed in a modal transfer matrix form in frequency domain and numerical results are presented to show the effects over wide ranges of frequency, viscosity and mean flow parameters on the propagation constants. The theoretical formulation allows for the duct walls to have finite impedance, but no numerical results are presented for lined ducts or ducts carrying a sheared mean flow.

1. Introduction

The interest on the effects of fluid viscosity and thermal conductivity on propagation of acoustic wave motion in uniform hard-walled ducts containing a homogeneous quiescent medium is classical. The wave equation for the problem is given by Kirchhoff [1] in frequency domain. His interest on the problem appears to have been raised because the consideration of the viscosity alone by Helmholtz did not explain accurately enough the experimental findings of Kundt about the frequency dependence of the propagation constants in narrow circular tubes containing a quiescent perfect gas (references to the work of Helmholtz and Kundt are given in Ref. [1]). Kirchhoff derived, for vanishing acoustic temperature fluctuations and particle velocity conditions at the duct walls, a dispersion equation for the axisymmetric modes in circular ducts and, assuming that the acoustic viscous and thermal boundary layers are very thin, presented an approximate solution for the fundamental mode. This solution, which is now known as the wide pipe solution, did not agree with Kundt's results in all detail, but it showed that the effect of heat conduction on the attenuation of sound in a perfect gas is of the same order as that of

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viscosity. The theory of Kirchhoff is described in detail by Rayleigh [2], who also proposed a solution for the fundamental mode for another extreme case of the problem, that is, very narrow pipes in which the acoustic boundary layers extend over the whole cross-section. Since these pioneering works, numerous approximate forms of Kirchhoff's dispersion equation have been proposed for evaluation of the fundamental propagation constants for specific frequency ranges and duct sizes. A review of the various approximations has been presented by Tijdeman [3]. More recent approximate studies are presented in Refs. [4–6] without mean flow and in Refs. [6–12] with mean flow.

However, the current knowledge on the higher order modes of Kirchhoff's dispersion equation and the non-axisymmetrical modes, which are not covered by it, is much less complete. A dispersion equation, which is claimed to encompass all modes in hard-walled circular ducts, has been proposed by Bruneau et al. [13–15] by neglecting the effect of the mean flow. The dispersion equation in Ref. [13] has an error; Ref. [14] gives the correct equation without derivation and elaborates on its approximate representations. The correct dispersion equation is presented in Ref. [15] with full derivation in time and frequency domains. But the authors did not present any numerical results in these studies. Liang and Scarton [16] presented some numerical results for the propagation constants of few higher order modes in hard-walled circular pipes without a mean flow, but the dispersion equation was not given and, for the parameters considered in the numerical examples, the viscothermal effect is negligibly small and the propagation constants appear to be barely discernible from their inviscid values. Also, the authors assume that the radial gradient of the acoustic temperature vanishes at the duct boundaries. The usual condition, which is used in the above cited previous studies, is the vanishing of the acoustic temperature. The rational for this is that, the small temperature fluctuations in the fluid cannot cause discernible changes in the temperature of the wall, because the corresponding heat transfer occurs to and from the wall periodically and, consequently, the boundary temperature remains practically the same as the mean temperature of the fluid [17].

The present paper aims to extend the classical theory [1] so that all higher order modes are encompassed with the effects of mean flow and finite wall impedance included for ducts containing real fluids. The analysis confirms the dispersion equation proposed in Ref. [15] and extends it for the effect of uniform mean flow and finite wall impedance. The solution of the convected wave equation is presented in multimodal transfer matrix form in frequency domain. It is shown that, for the more important axisymmetric modes in circular ducts, the propagation constants can have two uncoupled branches. The first of these branches is given by Kirchhoff's dispersion equation and includes the relatively well studied fundamental mode, which is until now considered to be the least attenuated mode. The second branch appears to have escaped attention of Kirchhoff [1] and also in all publications thereafter. The modes associated with the second branch become effective if the circumferential component of the particle velocity is non-trivial and can be less attenuated than the modes associated with the first branch if parameters are favorable. The paper gives numerical results showing the effects, over wide ranges, of the salient non-dimensional parameters representing frequency, viscosity and mean flow velocity, on the propagation constants associated with both branches for the fundamental and higher order axisymmetric modes, and also for the propagation constants of the non-axisymmetric modes in circular ducts. The results indicate the existence of modes akin to the 'strange modes' [18] observed in lined ducts if the parameters are favorable. Such modes are associated with instability waves. However, the observability of the strange modes in practical ducts is still a matter of debate and no attempt is made in this paper to study their evolution. The analysis allows for the presence of finite impedance walls, but no numerical results are presented for lined ducts.

In the contemporary studies [13,15,16], use is made of *a priori* decomposition of the acoustic particle velocity field into irrotational and solenoidal components. This decomposition is always possible for continuous vectors, however, the classical approach, in which this vector decomposition comes out as result, is preferred in the present analysis in order to maintain direct connection with the classical theory [1].

In this first time inclusion of the mean flow in the general theory, flow-acoustic interactions and turbulence diffusion are neglected, as these topics deserve separate studies. Also, the mean flow velocity profile is approximated by a uniform one having the same average velocity. This neglects the effect of refraction (see the next paragraph), but makes possible an analytical solution for the effect of convection, which is advantageous for insight into the physics of the problem. However, the assumption of uniform mean flow introduces a complication due to the requirement that a mean boundary layer should separate the duct walls from the uniform core flow. In the present analysis, the presence of the mean boundary layer is accounted for by invoking the Ingard–Myers theory [19]. This theory hypothesizes that the boundary layer is infinitely thin and that the particle displacement, as well as the acoustic pressure, are continuous across it. Auregan et al. [20] show that, in the absence of slip flow at the duct walls, the Ingard–Myers theory is adequately accurate if $\delta_A/\delta \ll 1$, where δ and δ_A denote, respectively, the thickness of the mean and acoustic boundary layers, the latter being assumed to be small compared to the wavelength. This condition is satisfied for the mean flow Mach numbers and frequency ranges that are of interest in most practical flow duct applications.

Whilst the effect of convection is relevant for all frequencies, the effect of refraction enters only at relatively high frequencies, and tends to increase with the gradient of the mean flow. For calculations, the mean flow velocity profile is usually modeled by using tractable distributions, however, based on the previous solutions for ducts containing an inviscid fluid, it is known that, the exact shape of the mean flow velocity profile is not important if the shape factor is about the same and the boundary layer is characterized by the displacement thickness [21]. On this premise, the refractive effect in a fully developed mean flow can be predicted approximately by assuming that the mean velocity profile is uniform over the duct section, except in a relatively thin boundary layer. The effect of refraction is then confined to the boundary layer and can be estimated by using the uniform mean flow solution presented in this paper, and deferring the boundary condition on the