Journal of Sound and Vibration 333 (2014) 2422–2431

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Proper Generalized Decomposition applied to linear acoustic: A new tool for broad band calculation

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A R T I C L E I N F O

Article history:
Received 30 May 2012
Received in revised form
7 January 2014
Accepted 8 January 2014
Handling Editor: R.E. Musafir
Available online 15 February 2014

A B S T R A C T

The calculation of the acoustic response of systems in frequency bands is becoming increasingly important in simulation-based engineering design. This is particularly true in medium-frequency bands, where the response is very sensitive to the frequency. Some standard techniques for addressing these problems present a frequency dependent formulation and may involve fixed-frequency calculations at many different frequencies. In this paper, we propose a new technique which combines the Variational Theory of Complex Rays (VTCR) with Proper Generalized Decomposition (PGD) and does not require the resolution of acoustic problems at many frequencies. In this approach, the VTCR is used to find an approximate solution of a medium-frequency acoustic problem using only a few degrees of freedom (DOFs). Then, PGD is used to find a representation of the approximate solution which is separated between two variables, the wave propagation direction and the frequency. A relevant numerical example is used to present the strategy and illustrate its applicability for frequency band calculations.
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1. Introduction

In many industrial contexts, such as aerospace applications or car design, numerical prediction techniques are being increasingly used because they limit the need for physical prototypes to a minimum. In these industries and in the specific case of acoustic problems, engineers are often interested in the response of systems in frequency bands. The efficient calculation of the acoustic behavior of systems is the topic of this paper.

Some numerical techniques to predict the acoustic behavior of systems in frequency bands require calculations at many different fixed frequencies. This natural and straightforward approach, even with an efficient numerical tool, can easily lead to prohibitive computation times and huge needs in terms of data storage. This is particularly true in the context of medium-frequency bands because in these ranges the response is very sensitive to the frequency, which requires a very fine frequency resolution and makes the problem even worse. Therefore, for medium-frequency acoustic problems, there is a clear need to improve the efficiency of the prediction techniques for frequency bands. Some techniques have already been developed in order to do that [1–3].

In this paper, we propose an alternative approach to the development of a medium-frequency prediction techniques in frequency bands which leads to a separated representation of the unknown field. This approach uses the Variational Theory

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http://dx.doi.org/10.1016/j.jsv.2014.01.014

of Complex Rays (VTCR) in combination with Proper Generalized Decomposition (PGD) [4,5] a promising model order reduction technique. The VTCR was introduced in [6] and belongs to the Trefftz family of methods which uses exact solutions of the governing differential equations for the expansion of the field variables.

The decisive advantage of all Trefftz methods is that since they use exact solutions of the governing equations, no refined spatial discretization inside the domain is necessary. Therefore, the model's size and the computational effort may be considerably less than with element-based methods. The VTCR differs from other Trefftz methods (such as the partition of unity method [7], the ultra-weak variational method [8], the least-squares method [9], the discontinuous enrichment method [10], the element-free Galerkin method [11], the wave boundary element method [12] or the wave-based method [13,14]) by the way it handles the transmission conditions at the inter-element boundaries and by the types of shape functions it uses. It has already been shown to be capable of finding accurate solutions of vibration problems involving 3D plate assemblies [15], plates with heterogeneities [16] and shell structures [17] as well as solutions of acoustic problems [18]. The VTCR is based on an original variational formulation of the problem which was developed in order to allow the approximations within elements Ω_e to be *a priori* independent of one another. Thus, in each element, any type of shape function can be used, as long as it verifies the governing Helmholtz equation. This property gives the approach great flexibility and, consequently, makes it very efficient because shape functions with a strong physical meaning related to the desired solution can be introduced without difficulty. Concerning PGD, this has been found to be an efficient technique for the resolution of multiparametric problems (problems which depend on many parameters such as the space and time problems, and the space and uncertain problems), which is what we need in order to deal with the difficulties of having to solve an acoustic problem for multiple frequencies. Therefore, even though PGD is compatible with any wave approach, a combination of PGD and the VTCR is an obvious choice when it comes to the resolution of problems in mid-frequency bands.

The VTCR has already been adapted to the handling of frequency bands in [19,20]. The authors proposed new algorithms for the calculation of multiple-frequency solutions, either by using a set of parameters to derive a discrete approximation of the frequency-dependent quantities within the VTCR matrix or by expanding the VTCR matrix and the right-hand side of the system to be solved into Taylor series with respect to the frequency. The objective of the third technique we are proposing here is to open a new path with regard to frequency band analysis.

The paper is structured as follows: Section 2 presents the problem being considered, which is a 2D acoustic problem driven by the Helmholtz equation. Section 3 reviews the principles of the VTCR, which leads to a discretized version of the problem. Section 4 introduces the use of PGD for frequency-dependent acoustic problems expressed in VTCR form. Section 5 illustrates the strategy in the case of a 2D acoustic problem. Conclusions and perspectives are presented in Section 6.

2. The reference problem

Let us consider the general 2D interior dynamics problem of a bounded acoustic domain Ω, filled with a fluid characterized by sound velocity c_0 and density ρ_0, to be studied in the frequency interval $I =]\omega_0 - \Delta\omega/2; \omega_0 + \Delta\omega/2[$, where ω_0 denotes the central frequency and $\Delta\omega$ the bandwidth of the frequency band being considered. The problem is to find $p(\mathbf{x}, \omega)$, $(\mathbf{x}, \omega) \in \Omega \times I$ such that

$$\left| \begin{array}{ll} \Delta p + k^2 p = 0 & \text{over } \Omega \times I \\ p = p_d & \text{over } \partial_p\Omega \times I \\ L_v(p) = v_d & \text{over } \partial_v\Omega \times I \\ p - ZL_v(p) = h_d & \text{over } \partial_Z\Omega \times I \end{array} \right. \tag{1}$$

where Δ is the Laplacian operator, $k = (1 - \mathrm{i}\eta)\omega/c_0$ is the wavenumber (η being the absorption coefficient), p_d a prescribed pressure, v_d a prescribed velocity, Z a given impedance and h_d a given function. Operator $L_v(\square)$ is defined by $L_v(\square) = (\mathrm{i}/\rho_0\omega)(\partial\square/\partial\mathbf{n}) = (\mathrm{i}/\rho_0\omega)\mathbf{n}^T\nabla(\square)$, $\mathbf{n}$ being the outward normal and ∇ the gradient operator. $\partial_p\Omega$, $\partial_v\Omega$ and $\partial_Z\Omega$ are the parts of the boundary $\partial\Omega$ of Ω where the pressure, the velocity and a Robin condition are respectively prescribed. The uniqueness of the solution of this reference problem is ensured by a strictly positive η.

If we introduce a partition of Ω into n_{el} non-overlapping elements Ω_e, the following additional equations must be verified in order to ensure the continuity of the solution and its normal derivative along $\Gamma_{e,e'} = \Omega_e \cap \Omega_{e'}$:

$$\left| \begin{array}{ll} p_e - p_{e'} = 0 & \text{along } \Gamma_{e,e'} \times I \\ L_v(p_e) + L_v(p_{e'}) = 0 & \text{along } \Gamma_{e,e'} \times I \end{array} \right. \tag{2}$$

3. The VTCR variational formulation of the reference problem

The VTCR variational formulation of reference problem (1), (2) requires the definition of the functional space of the functions which satisfy the homogeneous Helmholtz governing equation, *i.e.* the first equation in (1):

$$S^e = \{p_e; \Delta p_e + k^2 p_e = 0 \text{ over } \Omega_e \times I\} \tag{3}$$

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