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On variable length induced vibrations of a vertical string

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ABSTRACT

The purpose of this paper is to study the free lateral responses of vertically translating media with variable length, velocity and tension, subject to general initial conditions. The translating media are modeled as taut strings with fixed boundaries. The problem can be used as a simple model to describe the lateral vibrations of an elevator cable, for which the length changes linearly in time, or for which the length changes harmonically about a constant mean length. In this paper an initial-boundary value problem for a linear, axially moving string equation is formulated. In the given model a rigid body is attached to the lower end of the string, and the suspension of this rigid body against the guide rails is assumed to be rigid. For linearly length variations it is assumed that the axial velocity of the string is small compared to nominal wave velocity and the string mass is small compared to car mass, and for the harmonically length variations small oscillation amplitudes are assumed and it is also assumed that the string mass is small compared to the total mass of the string and the car. A multiple-timescales perturbation method is used to construct formal asymptotic approximations of the solutions to show the complicated dynamical behavior of the string. For the linearly varying length analytic approximations of the exact solution are compared with numerical solution. For the harmonically varying length it will be shown that Galerkin's truncation method cannot be applied in all cases to obtain approximations valid on long timescales.

1. Introduction

Many engineering devices are represented by axially moving continua. Translating media with constant length can model such low- and high-speed slender members as conveyor belts [1–4], chair lifts, power-transmission chains, pipes transporting fluids [5,6], aerial tramways, magnetic paper tapes, band saws, and transport cables. In many applications, systems including elevator cables [7–10], paper sheets [11], satellite tethers, flexible appendages, cranes and mine hoists [12,13], and cable-driven robots exhibit variable-length and transport speed during operation. The traveling, tensioned Euler–Bernoulli beam and the traveling flexible string are the most commonly used models for such types of axially moving continua. They are classified in the category of one-dimensional continuous systems and consequently the displacement field depends on time and on a single spatial coordinate. The last few decades have seen an extensive research effort on the dynamics of translating media, where most studies were restricted to cases with constant span length and transport velocity.

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Vibrations of horizontal and vertical translating strings and beams have been studied by many researchers. The forced response of translating media with variable length and tension was analyzed by Zhu and Chen [7]. The effects of bending stiffness and boundary conditions on the dynamic response of elevator cables were examined by Zhu and Xu in [8]. By transforming the governing partial differential equation to ordinary differential equations, Carrier [14] studied the response of a translating string with varying-length. Tabarrok et al. [15] studied the dynamics of a translating beam with varying-length and derived the equations of motion of a simple cantilever beam model utilizing Newton's second law. Among the earliest known considerations, for some special cases Vesnitskii and Potapov [16] found the exact solutions of one-dimensional mechanical systems of variable length. For earlier work on strings with mass–spring systems emulating an elevator, the reader is referred to Yamamoto et al. [17] and Terumichi et al. [18]. They assumed a constant transport velocity for translating strings. Chi and Shu [19] calculated the natural frequencies associated with the longitudinal vibration of a stationary cable and a car system. General stability characteristics of horizontally and vertically translating strings and beams with arbitrarily varying-length and with various boundary conditions were investigated by Zhu and Ni [10]. An active control methodology using a pointwise control force and/or moment was developed by Zhu et al. [9] to dissipate the vibratory energy of a translating medium with arbitrarily varying length.

To improve the design of elevators, one of the major tasks is to develop a better understanding of elevator cable dynamics and new methods to effectively reduce the vibration and noise. The dynamics of vertical media with variable length, velocity and tension is the subject of this paper. Due to small allowable vibrations the lateral and the vertical cable vibrations in elevators can be assumed to be uncoupled and only the lateral cable vibrations in elevators are considered here. The elevator car is modeled as a rigid body of mass m attached at the lower end of the cable, and the suspension of the car against the guide rails is assumed to be rigid, where external excitation is not considered at the boundaries. This is considered to be a basic and a simple model of an elevator cable from the practical viewpoint. The initial-boundary value problems will be studied, and the explicit asymptotic approximations of the solutions, which are valid on a long timescale, will be constructed as for instance described in [20,21]. Two cases for the varying-length will be considered, (i) $l(t)=l_0+\overline{v}t$, where l_0 is the initial cable length and $\overline{v}$ is the constant cable velocity, and (ii) $l(t)=l_0+\beta\ \sin(\omega t)$, where β defines a length variation parameter and ω signifies the angular frequency of length variation, and $l_0>|\beta|$. For both cases of varying-length different dimensionless parameters will be used to obtain dimensionless equations of motion. For the first case, it is assumed that $\overline{v}\sqrt{\rho/mg}=\mathcal{O}(\varepsilon)$ and $\rho L/m=\mathcal{O}(\varepsilon)$, where ρ is the cable mass density, m is the car mass, g is the acceleration due to gravity, and L is the maximum length of the cable. For this case, the exact solution of the initial-boundary value problem has been approximated up to $\mathcal{O}(\varepsilon)$ and the free response of the elevator system is obtained in closed form solutions. For the second case, it is assumed that $\beta/L=\mathcal{O}(\varepsilon)$ and $\rho L/(m+\rho L)=\mathcal{O}(\varepsilon)$, where L is the maximum cable length and $L \gg |\beta|$. For this case, it will be shown that Galerkin's truncation method cannot be applied for the parameter $|\alpha|\le 2$ due to the distribution of energy among all vibration modes. To our knowledge, the explicit construction of approximations of oscillations for these types of problems has not been given before.

The outline of the paper is as follows. In Section 2, the governing equations of motion are established. In Sections 3 and 4, a two-timescales perturbation method is applied to construct formal asymptotic approximations for the solutions of the initial-boundary value problems. It turns out for the case with the harmonically varying length that there are infinitely many values of ω that can cause internal resonances. In this paper only the resonance case $\omega=\pi/l_0$ is investigated and also a detuning case for this value is studied. Finally, in Section 5, some remarks are made and some conclusions are drawn.

2. The governing equations of motion

The vertically translating cable in elevators has no sag and will be modeled as a taut string with fixed boundaries in the horizontal direction, as shown in Fig. 1. The elevator car is modeled as a rigid body of mass m attached at the lower end $x=l(t)$, and suspension of the car against the guide rails is assumed to be rigid. During its motion the cable of density ρ has a variable length $l(t)$ and an axial velocity $\overline{v}(t)=\dot{l}(t)$, where the over dot denotes time differentiation. The cable is assumed to be inextensible with an arbitrarily prescribed translational velocity $\overline{v}(t)$, where t is the time. A positive or negative transport velocity designates extension or retraction of the cable, respectively. The lateral and longitudinal vibrations of elevator cable are assumed to be uncoupled. In this paper longitudinal vibrations will not be considered. Relative to the fixed coordinate system as shown in Fig. 1, the lateral displacement of the cable particle instantaneously located at spatial position x at time t, where $0\le x\le l(t)$, is described by $u(x,t)$.

The equations of motion for vertical string with variable length, velocity and tension can be obtained by using extended Hamilton's principle [10], and are given by

$$\rho\frac{D^2u(x,t)}{Dt^2}-\frac{\partial}{\partial x}\left(P(x,t)\frac{\partial u(x,t)}{\partial x}\right)=0,\quad t>0,\ 0<x<l(t), \tag{1}$$

$$u(0,t)=u(l(t),t)=0,\quad t>0, \tag{2}$$

$$u(x,0)=f(x)\quad\text{and}\quad u_t(x,0)=h(x),\ \ 0<x<l(0), \tag{3}$$