



Analysis of thermoelastic dissipation in circular micro-plate resonators using the generalized thermoelasticity theory of dual-phase-lagging model



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ABSTRACT

This study investigates the thermoelastic dissipation of micro-plate resonators by using the generalized thermoelasticity theory of dual-phase-lagging model. Explicit formulae of thermoelastic damping and frequency shift are derived. Influences of the plate thickness and vibration frequency on the thermoelastic damping are examined. Phenomena distinct from those of classical theory are observed in the numerical results of thermoelastic damping in micro-plate resonators. These results may bring new insights into the study of thermoelastic damping at submicrometer or nanometer scale.

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1. Introduction

Micro- and nano-electromechanical resonators are one of the important types of micro-electro-mechanical system (MEMS) and nano-electro-mechanical system (NEMS) devices. They have been widely applied in various technological fields such as ultrasensitive mass detection, mechanical signal processing, scanning probe microscopes, etc. [1,2]. Energy dissipation due to different damping factors in micro-resonators is particularly significant because it is closely related to the performance and precision of micro-resonators based devices [2,3]. Thus, it is very important to have an in-depth understanding on the energy dissipation mechanisms and a thorough analysis of their influences on the operation of micro-resonators.

Among several energy dissipation mechanisms, thermoelastic damping has been particularly identified as a major cause of energy dissipation for a large range of micro/nano-mechanical resonators [4]. Since analyzing and predicting thermoelastic damping are crucial to the development of MEMS/NEMS resonators with high quality factor, a great deal of work has been conducted on this topic. Lifshitz and Roukes [3] derived a formula of quality factor (Q-factor) for thermoelastic damping in micro-beam resonators based on the classical coupled thermoelasticity theory of Fourier thermal conduction model. Their work brought much attention of the academic community to this topic. A lot of work has been devoted to the thermoelastic damping in nano- and micro-resonators following Lifshitz and Roukes' work. Guo and Rogerson [5] presented two dimensional analyses of the frequency shift of a micro-beam resonator due to thermoelastic coupling effect by considering heat conduction along both the beam thickness and beam span. Prabhakar and Vengallatore [6] employed Galerkin method to calculate the frequency shifts and thermoelastic damping in micromechanical resonators by considering two-dimensional heat conduction. Hao et al. [7]

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developed a thermal-energy method for calculating thermoelastic damping. Their method is based on the dissipation of vibration energy is permanently converted into thermal energy and thermoelastic damping can thus be calculated by seeking the generation of thermal energy per cycle of vibration. Thermal-energy method is more computationally efficient than the commonly employed complex-frequency method. Furthermore, Ru [8] studied the influences of surface stress on the thermoelastic dissipation of nanowire resonators. Recently, Tunvir and Ru [4] examined the effect of cross-sectional shapes on thermoelastic dissipation of micro/nano-beam resonators.

Circular plates are a common structural type in many sensors and resonators. Hao [9] adopted the thermal-energy method to analyze thermoelastic damping of in-plane vibration in a circular plate resonator. Sun and Tohmyoh [10] studied thermoelastic damping of axisymmetric flexural vibration in circular plate resonators. Sun and Saka [11] further investigated the thermoelastic damping of general vibration modes in circular resonators in the context of coupled theory of thermoelasticity. Their formula of thermoelastic damping for micro-plate resonators is different from that of Lifshitz and Roukes only by a factor $\kappa = (1 + \nu)/(1 - 2\nu)$, in which ν is Poisson's ratio.

Li et al. [12] also employed the thermal energy method to study thermoelastic damping in rectangular and circular micro-plate resonators by calculating the energy dissipated per cycle of vibration over the volume of a micro-plate. Their conclusion is that the strain component in the plate thickness direction cannot be neglected in calculating the cubical dilatation. However, one of the basic assumptions in Love–Kirchhoff thin plate theory is that the strain component in the plate thickness direction is negligible [13]. Love–Kirchhoff theory would not stand without this assumption. Therefore, it is of little significance to study the effect of nonzero strain component in the plate thickness direction within the framework of Love–Kirchhoff thin plate theory.

All works based on the classical theory of thermoelasticity predict that the existence of a peak in thermoelastic damping, which occurs around micrometer scale. When the size of micro-resonators goes up to more than 100 μm or down to nanometer scale, thermoelastic damping will drop along with the increase or decrease of the size of resonators. However, experimental results show that device miniaturization beyond the submicron scale leads to increased energy dissipation and Q-factor is roughly proportional to $V^{1/3}$ over a large size range, from macroscopic scale down to nanometer scale, in which V is the volume of resonators [14].

The classical theory of thermoelasticity, in which the equation governing the temperature distribution is of parabolic type, therefore implies infinite speed of propagation of thermal disturbances [15]. On the other hand, heat conduction in MEMS and NEMS devices demonstrates lots of distinct phenomena such as the size effect and wave phenomena, which cannot be captured by the classical Fourier law [16]. Therefore, it is of great importance to establish non-Fourier heat conduction models accounting for the wavelike phenomenon occurring in the microscale heat conduction. Several nonclassical theories have been formulated in the literature such as Lord and Shulman's model, Green and Lindsay's model and dual-phase-lagging model [17,18].

Some researchers have adopted these nonclassical theories to study thermoelastic damping in micro-resonators. Sun et al. [19] analyzed thermoelastic damping of a beam resonator. Sharma [20] investigated thermoelastic damping and frequency shift in micro/nanoscale anisotropic beams. Sharma and Sharma [21] studied damping in micro-scale circular plate resonators. They all employed the generalized theory of thermoelasticity of Lord and Shulman's model. These studies all predict that the thermoelastic damping varies with the size of micro-resonators in the same way as the study by the classical theory predicts except that there are some differences in the maximum value of thermoelastic damping.

In this work, we attempt to investigate the thermoelastic damping effect of a circular micro-plate resonator under two types of boundary conditions by using a nonclassical theory of thermoelasticity. The dual-phase-lagging heat conduction model, which has been demonstrated to be admissible to the second law of extended irreversible thermodynamics and Boltzmann transport equation [22], is employed to formulate a generalized (non-Fourier) thermoelasticity theory. Explicit formulae of thermoelastic damping and frequency shift are derived. The results of the present work are compared with those of classical theory. Distinct relationships between the thermoelastic damping and the plate thickness or vibration frequency are observed in the numerical results. We believe that this work may bring new insights into the study of thermoelastic damping at the submicrometer or nanometer scale.

2. Formulation of the problem based on the nonclassical thermoelasticity theory of dual-phase-lagging model

This study is concerned with the thermoelastic damping effect in a circular micro-plate resonator of uniform thickness h and radius R . A cylindrical coordinate system is employed with its origin at the center of the plate. The (r, θ) plane is kept on the neutral surface of the plate and the z axis is perpendicular to the neutral surface. The micro-plate resonator is assumed to be in equilibrium initially with a uniform environmental temperature T_0 and free of deformation and stress. It undergoes flexural vibrations of small amplitude, and the deformation and motion can be described by the Kirchhoff–Love plate theory. The equation of motion of the micro-plate, with the incorporation of thermal moment due to temperature change, is given by [11,21]

$$D\nabla^4 w + D(1 + \nu)\alpha_T \nabla^2 M_T + \rho h \frac{\partial^2 w}{\partial t^2} = 0, \quad (1)$$

where w is the flexural displacement of the plate; ρ , ν and α_T are the density, Poisson's ratio and thermal expansion coefficient, respectively; $D = Eh^3/12(1 - \nu^2)$ is the plate flexural rigidity; E is Young's modulus; $M_T = (12/h^3) \int_{-h/2}^{h/2} \theta z \, dz$ is the

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