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Identification of stiffness and damping properties of plates by using the local equation of motion

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ABSTRACT

This paper deals with the identification of stiffness and damping properties of vibrating structures by an inverse method inspired from the Force Analysis Technique (FAT). The proposed approach uses a local equation of motion assumed a priori, which provides a relative straightforward relationship between the displacement field and material properties. The spatial derivatives of the displacement in the equation are calculated using finite differences. As this operation amplifies measurement noise, a regularization step is applied before solving the inverse problem. A procedure is proposed to automatically adjust the level of regularization. The method also allows one to identify local stiffness and damping on a heterogeneous structure. Illustrations for both homogeneous and heterogeneous cases are shown using simulated and measured displacement fields.

1. Introduction

Composite materials, which provide high stiffness and light weight, are increasingly used in the aerospace and automotive industry. During the conception of structures including such materials, an extensive use of finite element models is made to predict the dynamic and vibroacoustic behavior. To achieve predictions that are as accurate as possible, one of the key challenges is to enter the right material characteristics into the model. As a wide variety of composite materials exist, their elastic and damping characteristics are rarely available in tables and must be obtained experimentally. Moreover, if the finite element method uses plate elements, the determination of the characteristics of the equivalent plate (stiffness and damping for all types of loads) needs the choice of kinematic assumptions. This choice influences the results of the model and has to be confirmed by measurements. Measurement techniques based on modal analysis theory, which is well established [1], are widely used. The measurement of modal parameters (natural frequencies, mode shapes and damping ratio) on a test structure, generally combined to a numerical model, allows one to identify elastic and damping properties of the constitutive material [2]. However, there are several limitations to such methods. First, as it is a global approach, it is not possible to characterize the material directly on the target structure itself: experiments are generally carried out on specific test specimens, a precise knowledge of boundary conditions being crucial. Secondly, modal parameters are difficult to measure in the mid-frequency domain, characterized by a strong modal overlap. Because the stiffness and damping of composites may strongly vary with frequency, it is necessary to obtain material properties in a wide frequency range instead of extrapolating results obtained at low frequencies.

In recent years, works have been done on the estimation of wavenumbers in a structure, from which material properties such as loss factor or dynamic stiffness may be deduced [3–6]. Such wave-based approaches overcome most limitations

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of the modal approach. In particular, they are applicable at any frequency and do not require any knowledge about boundary conditions. Their principle is to find the wavenumber giving the best fit between the displacement field measured at a given driving frequency and a theoretical field consisting of a sum of waves. This theoretical field can be viewed as the solution of an equation of motion involving material properties, which may be related to the identified wavenumber.

This paper presents a new method to identify material properties from the displacement field measured locally on a structure. The underlying idea is to use the local equation of motion itself, which provides a relative straightforward relationship between the material properties and the displacement field. The method is inspired from the Force Analysis Technique (FAT, also known by the french acronym RIFF), initially aimed at identifying dynamic loads acting on structures [7–11]. In the Force Analysis Technique, the measured displacement field is injected into a discrete form of the local equation of motion, in which the spatial derivatives are approximated by a finite-difference scheme, leading to a straightforward calculation of the force acting in the area considered. This simple principle, combined with an appropriate regularization procedure (windowing and filtering), has been successfully applied to beams [7], plates [8,9], shells [12,13], and thereafter extended to more complex structures by using a finite element operator instead of an analytical equation of motion [14]. To localize and quantify external forces, the Force Analysis Technique obviously requires the a priori knowledge of material properties involved in the equation of motion.

The originality of the present method is to identify unknown material properties from the equation of motion by considering an area of the structure which is free of any external force. The key ingredients for the identification of the material properties (approximation of partial derivatives, regularization) are the same as those used in FAT, complemented with specific developments to compensate the lack of a priori knowledge about the structure. In particular, a novel automatic regularization technique is proposed. The paper focuses on the application to plates. However, the method may be extended to other types of structures for which a differential equation exists.

The continuation of this paper is structured as follows. First, the general principle of the method is exposed more in detail. Then, the feasibility of the proposed approach is assessed using simulated displacement fields. The effect of noise is illustrated and a procedure to automatically adjust the regularization is described. The applicability of the method is assessed for both homogeneous and heterogeneous structures. The last part presents experimental results.

2. Principle of the identification technique

The equation of motion of an isotropic thin plate in the harmonic regime is [15]

$$D\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)-\rho h\omega^2 w(x,y)=f(x,y), \tag{1}$$

where D is the flexural stiffness, ρ the density, h the thickness, ω the angular frequency, $w(x,y)$ the transverse displacement and $f(x,y)$ the force per unit area. The material is assumed to be linear viscoelastic. Therefore, the flexural stiffness

$$D=\frac{E(1+\mathrm{j}\eta)h^3}{12(1-\nu^2)} \tag{2}$$

involves a complex Young's modulus $E(1+\mathrm{j}\eta)$, where j is the unit imaginary number and η denotes the loss factor, which characterizes material damping. The equation of motion (1) is local and therefore valid everywhere in the structure. More particularly, considering an area on which no external force is applied, i.e., $f(x,y)=0$, Eq. (1) becomes

$$\frac{D}{\rho h\omega^2}\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)=w(x,y). \tag{3}$$

Eq. (3) simply expresses the fact that the bilaplacian $\nabla^4 w(x,y)$ equals the displacement $w(x,y)$ up to a constant multiplier. The constant multiplier $D/\rho h\omega^2$ depends on the characteristics of the plate, and on the angular frequency ω. It should be noted that the quantity $D/\rho h$ is complex and may vary with frequency. It contains information about the stiffness and damping of the material. Thus, assuming that the displacement $w(x,y)$ is known, it seems possible to determine the quantity $D/\rho h$ from Eq. (3), at any frequency, without any knowledge about the exact location and amplitude of the external force, or about the boundary conditions. The only requirement is that no external force is applied at the considered location (x,y).

In practice, however, there are several obstacles to this simple idea. The first one is due to the presence of fourth-order partial derivatives in Eq. (3). Whereas the transverse displacement at a given location may be easily obtained by using an accelerometer or a laser vibrometer, its partial derivatives are less straightforward to obtain. To overcome this difficulty, one may measure the displacement at discrete abscissas (x_i, y_i) over a regular meshgrid, so that the partial derivatives may be approximated by a finite difference scheme. A discretized form of Eq. (3) is

$$\frac{D}{\rho h\omega^2}\left(\delta_{i,j}^{4x}+2\delta_{i,j}^{2x2y}+\delta_{i,j}^{4y}\right)=w_{i,j}. \tag{4}$$

The expressions of $\delta_{i,j}^{4x}$, $\delta_{i,j}^{4y}$ and $2\delta_{i,j}^{2x2y}$ may be found in Appendix A. The approximation of the partial derivatives introduces a bias which is likely to affect the value of $D/\rho h$ calculated from Eq. (4). Note that the corrected finite differences scheme proposed in [11] can also be applied.