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Stochastic interval analysis of natural frequency and mode shape of structures with uncertainties

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ABSTRACT

In this paper, natural frequencies and mode shapes of structures with mixed random and interval parameters are investigated by using a hybrid stochastic and interval approach. Expressions for the mean value and variance of natural frequencies and mode shapes are derived by using perturbation method and random interval moment method. The bounds of these probabilistic characteristics are then determined by interval arithmetic. Two examples are given first to illustrate the feasibility of the presented method and the results are verified by Monte Carlo Simulations. The presented approach is also applicable to solve pure random and pure interval problems. This capability is demonstrated in the third and fourth examples through the comparisons with the peer research outcomes.

1. Introduction

Realistically modeling uncertainties in determination of natural frequencies and mode shapes is crucial for studying characteristics of dynamic systems [1]. Stochastic methods have been well developed and become the most widely accepted ones for quantifying uncertainties when sufficient statistical information is available [2]. In these methods, uncertain inputs are modeled as random variables/fields/processes, and characteristics of uncertainties are described by mean value, variance and probability density function (PDF) etc. Probabilistic features of outputs are then obtained through uncertainty propagation [3]. Natural frequencies of dynamic systems with random parameters are often determined by solving the random eigenvalue problems [4]. Boyce [5] and Collins [6] did early research on this topic in sixties of last century. Scheidt and Purker [7] systematically studied random eigenvalue problems in their book. Various approaches [8–13] have been introduced to solve the random eigenvalue problems such as direct Monte Carlo Simulation method [8], perturbation method [9,13], random factor method [11] and asymptotic integral based method [12] etc.

However, in the case of lack of trustworthy statistical information, selection of proper PDF can lead to subjective results [14] in stochastic approaches. In these circumstances, interval method was introduced as an alternative to quantify these uncertainties with possible value range between crisp bounds without additional information concerning variations in their intervals [15]. Interval eigenvalue problem [16] has attracted more and more attention in the past two decades. Hollot and Bartlett [17] investigated eigenvalues of interval matrices. Chen et al. [18] proposed perturbation method for computing the bounds of eigenvalues of vibration systems with interval parameters. In this paper, first-order and second-order perturbation methods were presented. Qiu et al. [19] introduced vertex theorem to compute eigenvalue bounds of structures with uncertain-but-bounded parameters. Gao [20] proposed interval factor method to conduct interval analysis

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of natural frequency and mode shape of structures with interval parameters. Modares et al. [21] presented an element-by-element formulation to account for interval eigenvalue problem. Angeli et al. [22] investigated natural frequency intervals for systems with polytopic uncertainty. Fuzzy set theory [23] is another alternative method for quantifying uncertainties by using membership functions which describe the degree of possibility with which the uncertain quantities may take on the associated values [15]. Through α-level strategy, interval analysis indeed forms the core of fuzzy analysis [24]. In other words, a fuzzy analysis can be converted into a series of interval analyses on α-level levels.

Numerous researches on uncertain problems in determining structural dynamic characteristics have been carried out by using single type of uncertain model. However, in real engineering problems, it is very common that a large number of design variables and parameters exist in a system simultaneously. Some of them may have sufficient statistical information while the others may not. In these circumstances, stochastic and interval models are required at the same time. Various attempts have been made on mixed uncertain problems in static problems [25–30]. Based on first-order perturbation method and moment method, Gao et al. [25,26] proposed random interval moment method to analyze static response of structures with mixed random and interval properties. Du et al. [27] investigated reliability based design of structures by considering a mixture of random and interval variables using optimization technique. Qiu et al. [28] introduced the interval arithmetic into conventional reliability theory. Gao et al. [29,30] incorporated random interval moment method with Monte Carlo Simulation and Quasi Monte Carlo Simulation to study static response and reliability of structures with mixed uncertainties. In the contrast to extensive research on static mixed uncertainty problems, dynamic problems of structures with mixed random and interval variables have not been well addressed. In this paper, natural frequencies and mode shapes of structures with mixed random and interval parameters are investigated by using a hybrid stochastic and interval approach. Expressions for the mean value and variance of natural frequencies and mode shapes are derived by using the first-order perturbation method and random interval moment method [25]. The bounds of these probabilistic characteristics are then determined by interval arithmetic. The presented method is also capable to solve pure random and pure interval problems. Four numerical examples are given to illustrate applications of the presented method. Two examples are presented first to demonstrate the feasibility of the presented method and the results are verified by Monte Carlo Simulations. The capabilities for solving pure random and pure interval problems are also illustrated through the comparison with other two published examples.

2. Random interval moment method

Random interval moment method is briefly introduced here and more details can be found in Ref. [25]. Let $X(R)$ be the set of all real random variables on a probability space Ω. x^R is a random variable of $X(R)$. R denotes the set of all real numbers. $\overline{x^R}$, $\mathrm{Std}(x^R)$ and $\mathrm{Var}(x^R)$ are the mean value, standard deviation and variance of x^R respectively. y^I is an interval variable of $I(R)$ which denotes the set of all the closed real intervals. A list of notations is attached in Appendix A for convenience:

$$y^I = [\underline{y}, \overline{y}] = \{t, \underline{y} \le t \le \overline{y} | \underline{y}, \overline{y} \in R\} \tag{1}$$

$$y^c = \frac{\underline{y}+\overline{y}}{2}; \quad \Delta y = \frac{\overline{y}-\underline{y}}{2}; \quad \Delta y^I = [-\Delta y, +\Delta y] \tag{2}$$

where $\underline{y}$, $\overline{y}$, y^c, Δy and Δy^I are the lower bound, upper bound, midpoint value, maximum width and uncertain interval of y^I respectively.

Z^{RI} is defined as a function of a random vector $\overrightarrow{\mathbf{X}^R} = (x_1^R, x_2^R, \ldots, x_n^R)$ and an interval vector $\overrightarrow{\mathbf{Y}^I} = (y_1^I, y_2^I, \ldots, y_m^I)$. The mean value of $\overrightarrow{\mathbf{X}^R}$ is $\overline{\overrightarrow{\mathbf{X}^R}} = (\overline{x_1^R}, \overline{x_2^R}, \ldots, \overline{x_n^R})$ and midpoint value of $\overrightarrow{\mathbf{Y}^I}$ is $\overrightarrow{\mathbf{Y}^c} = (y_1^c, y_2^c, \ldots, y_m^c)$. Applying Taylor expansion at $(\overline{\overrightarrow{\mathbf{X}^R}}, \overrightarrow{\mathbf{Y}^c})$, the following expression can be acquired:

$$\begin{aligned} Z^{RI} &= f(\overrightarrow{\mathbf{X}^R}, \overrightarrow{\mathbf{Y}^I}) \\ &= f(\overline{\overrightarrow{\mathbf{X}^R}}, \overrightarrow{\mathbf{Y}^c}) + \sum_{j=1}^{m} \left\{ \frac{\partial f}{\partial y_j^I}\bigg|_{\overline{\overrightarrow{\mathbf{X}^R}}, \overrightarrow{\mathbf{Y}^c}} \right\} \Delta y_j^I \\ &+ \sum_{i=1}^{n} \left\{ \frac{\partial f}{\partial x_i^R}\bigg|_{\overline{\overrightarrow{\mathbf{X}^R}}, \overrightarrow{\mathbf{Y}^c}} + \sum_{j=1}^{m} \left\{ \frac{\partial^2 f}{\partial x_i^R \partial y_j^I}\bigg|_{\overline{\overrightarrow{\mathbf{X}^R}}, \overrightarrow{\mathbf{Y}^c}} \right\} \Delta y_j^I \right\} (x_i^R - \overline{x_i^R}) + \mathrm{Re} \end{aligned} \tag{3}$$

Ignoring the remainder term Re, the mean value and variance of the random interval function Z^{RI} can be computed by using moments of linear function [31] as

$$\mu(Z^{RI}) = E\left[Z^{RI}\right] = f(\overline{\overrightarrow{\mathbf{X}^R}}, \overrightarrow{\mathbf{Y}^c}) + \sum_{j=1}^{m} \left\{ \frac{\partial f}{\partial y_j^I}\bigg|_{\overline{\overrightarrow{\mathbf{X}^R}}, \overrightarrow{\mathbf{Y}^c}} \right\} \Delta y_j^I \tag{4}$$

$$\mathrm{Var}(Z^{RI}) = E\left[(Z^{RI} - \mu(Z^{RI}))^2\right]$$