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Vibration of a beam on continuous elastic foundation with nonhomogeneous stiffness and damping under a harmonically excited mass

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ABSTRACT

In this paper, a method of analysis of a beam that is continuously supported on a linear nonhomogeneous elastic foundation and subjected to a harmonically excited mass is presented. The solution is obtained by decomposing the nonhomogeneous foundation properties and the beam displacement response into double Fourier summations which are solved in the frequency–wavenumber domain, from which the space–time domain response can be obtained. The method is applied to railway tracks with step variation in foundation properties. The validity of this method is checked, through examples, against existing methods for both homogeneous and nonhomogeneous foundation parameters. The effect of inhomogeneity and the magnitude of the mass are also investigated. It is found that a step variation in foundation properties leads to a reduction in the beam displacement and an increase in the resonance frequency for increasing step change, with the reverse occurring for decreasing step change. Furthermore, a beam on nonhomogeneous foundation may exhibit multiple resonances corresponding to the foundation stiffness of individual sections, as the mass moves through the respective sections along the beam.

1. Introduction

The model of a beam on an elastic foundation has been extensively used over the years for modelling the dynamic behaviour of railway tracks [1–3]. Diverse solutions have been presented for this problem for varying types of loads; static and dynamic, stationary and moving, deterministic and stochastic. In their solutions, most authors assume that the beam is supported on a linear elastic foundation with homogeneous stiffness and damping parameters. Solutions to the homogeneous case are readily available in the literature, e.g. the Fourier and Laplace transformation methods as well as direct approaches of solving the differential equations. Although this fundamental assumption of homogeneous parameters provides good understanding of the dynamic behaviour of a beam on an elastic foundation, it limits the true representation of the practical situation in most railway based applications. This is because there are many practical instances of nonhomogeneity in railway tracks. For example, in Hunt [4] several classes of inhomogeneity/roughness in railway tracks including variations in track-bed profile, foundation stiffness, sleeper spacing are given.

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The effect of spatial variation of track stiffness on the vibration of infinite beams on elastic foundation has been studied by several authors, mostly with the use of perturbation techniques for relatively small variations. Mahmoud and Tawil [5] analysed the quasi-static response of beams on random elastic foundation using truncated power series expansion of the random displacements. Dynamic effects were later accounted for by Frýba et al. [6] who used a first-order perturbation technique with stochastic finite elements to obtain the steady-state solution of an infinite beam on a random foundation with uncertain damping, subjected to a constant moving force. The results, which are given in the form of variances of the deflection and bending moment of the beam, show that the coefficient of variation of deflection is larger than that of the bending moment at the point of application of the force, with randomness of the foundation stiffness being of greater significance than the uncertainty in the damping. Andersen and Nielsen [7] analysed an infinite beam resting on a Kelvin foundation with the inclusion of a shear layer and subjected to a moving SDOF vehicle. The spatial variation of the foundation vertical stiffness is described by a stochastic homogeneous field consisting of small random variations around a predefined mean value. A first-order perturbation analysis was proposed to establish the relationship between the variation of the spring stiffness and the responses of the vehicle mass and the beam. The accuracy of this method, however, depends on the speed of the vehicle as well as the degree of variation in the random track stiffness, with fairly poor results obtained for speeds and stiffness variations over 30 percent. Verichev and Metrikine [8] studied the instability of a mass moving along a beam that is supported on an inhomogeneous elastic foundation with periodically varying stiffness. Perturbation analysis was used to obtain analytic expressions for the vibration conditions of the beam to become unstable. In all these cases, only small variations in track stiffness and damping have been considered by the authors in order to guarantee the accuracy of their solutions when making use of the perturbation technique. For example, in [6], the coefficients of variation in stiffness and damping are kept small enough compared to unity; i.e. $|\varepsilon| \ll 1$ and $|\gamma| \ll 1$ respectively. The same assumption is also emphasised in [8] where the small parameter $\mu \ll 1$, for the same reason stated above. Also, most of these models only considered variation in stiffness whereas the damping in treated as constant. However, large levels of inhomogeneity can be present in railway track supporting structures; e.g. moving from one track type to another as from a ballasted track to a bridge structure or vice versa.

Other methods have also been employed in studying beams on nonhomogeneous foundations. Pavlović and Wylie [9] investigated the free vibration of a beam on Winkler foundation with linearly varying modulus along the beam span using a power series approach. They concluded that the free response is mainly divided into two regions; up to a certain value of the stiffness below which the response of the beam can be determined by averaging the linearly varying stiffness and adopting equivalent homogeneous models and the region beyond this value in which this cannot be done. Wave propagation in a beam on a Winkler foundation with random spatial variation of spring stiffness has been studied by Schevenels et al. [10] with the focus on understanding the influence of correlation length. Their results show that even small spatial variations can have an influence on the response at large distance from the source if the correlation length and the wavelength are of the same order of magnitude.

Generally, there are two categories of methods used in solving the differential equation governing the dynamic behaviour of a beam on elastic foundation. The first category is based on discretisation techniques such as the finite element and finite difference methods carried out in the space–time domain, whereas the second group adopts transformation techniques in the frequency and/or wavenumber domains; see e.g. Knothe and Grassie [11]. Advances in discretisation methods have been immense with applications to solving beams on linear homogeneous foundations; see e.g. [12], nonhomogeneous foundations; see e.g. [13,14], nonlinear foundations; see e.g. [15], etc. However, the applications of frequency–wavenumber domain methods have been limited to linear homogeneous problems or periodically nonhomogeneous ones, see e.g. [16]. It is intended in this work to extend the applications of frequency–wavenumber domain technique to incorporate other forms of nonhomogeneity in the differential equation.

In this paper, an alternative approach is proposed for analysing a beam on elastic foundation with nonhomogeneous stiffness and damping under a moving harmonically excited mass. The method addresses some of the limitations of the aforementioned methods as large levels of variation in both foundation stiffness and damping are considered. The effects of nonhomogeneity on the beam response are analysed. In Section 2, the model is presented together with the generalised differential equation describing the beam dynamic behaviour, and the proposed method of solution, involving the use of Fourier sums is presented in Section 3. The method is applied to railway tracks with continuous elastic foundation with step variation in properties in Section 4. Results are then presented in Section 5, including validation of the current method against existing methods such as the Fourier transformation method for homogeneous parameters and also standard finite elements approach in the space–time domain.

2. Model formulation

Fig. 1(a) shows an infinitely long Euler–Bernoulli beam supported on a continuous linear elastic foundation resting on a rigid base. The vertical foundation stiffness and damping are modelled, using springs and dashpots respectively, as generally nonhomogeneous in the spatial domain along the length of the beam. The beam is traversed by a vehicle, excited by an oscillating load, at constant velocity in the direction shown.

The vehicle is represented only by a mass, assumed to be a simplification of a train in which only the unsprung part is included, i.e. assuming the suspensions isolate the dynamics of the sprung components of the vehicle in the frequency range