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Study on the validity region of Energy Finite Element Analysis

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ABSTRACT

The validity domain of Energy Finite Element Analysis (EFEA) is studied in this paper. The validity region and criterion of EFEA are studied theoretically from the formation of reverberant plane wave field, one of the main assumptions of EFEA. The studies are acquired by virtue of the equation of radiative energy transfer method, a similar wave method that can express the direct field and its conversion relationship with reverberant field exactly. The result shows that the SEA criterion of diffuse field derived by Le Bot can be used as a good indicator for the EFEA validity. Numerical simulations on a rectangular plate with different physical parameters are applied to validate the criterion. The validity region and the diagrams of validity of EFEA are assessed and discussed. Some noteworthy conclusions about EFEA are drawn.

1. Introduction

In the field of high frequency structural response prediction, the use of classical approaches like Finite Element Method (FEM) is time-consuming and requires very high computational costs. Alternatively, Statistical Energy Analysis (SEA) is a widely accepted tool with considerable industrial applications. However, SEA provides only a lumped estimation of vibration energies averaged over defined subsystems and cannot provide the spatial distribution inside subsystems. Energy Finite Element Analysis (EFEA) is a recent method for high-frequency structural response prediction. In EFEA, the structural response is described by energy variables in wave-based differential formulation, and can be numerically solved by Finite Element Method. Compared with SEA, EFEA is capable of predicting the spatial variation inside vibrating subsystems, and the local effects such as local damping and non-uniform distributed loading can be modeled. This method has been successfully applied to structural and acoustical components such as bars, beams, membranes, plates, shells, acoustic enclosure, actual built-up structures and vibro-acoustics problems with increasing popularity [1–6].

Despite the extension of theoretical studies on EFEA and its widespread applications in engineering, limitations of underlying theories emerged with the increasing difficulties in satisfying the EFEA assumptions in actual engineering. These difficulties motivate basic studies to better understand EFEA itself along with its limitations. The assumptions, approximations, limitations and the theoretical basis in the derivation of fundamental EFEA governing equation limit its validity region and criteria. In addition, the increasing popularity of EFEA in engineering makes applicability, validity and the choice of modeling methods crucial. Therefore, studies on the validity region and criterion of EFEA itself are of significant importance in the theoretical development and engineering application of EFEA.

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But until now, compared to its theoretical development and popularity in engineering, the studies on validity regions and criterion of EFEA still lag behind; only a little literature can be found, some of which might be vague, non-generally applicable or theoretically flawed. Gur et al. [7] proposed a wavelength criterion of the validity of EFEA for the plates and beams of Ford automobile structures. The results show that, the EFEA is capable of analyzing vibrations in automotive structures in the frequency range from 80 Hz to 250 Hz for plates with characteristics lengths greater than approximately 0.9 m only; the characteristic plate length should be 2.4 times greater than the bending wavelength for the lowest frequency of interest. Vlahopoulos [8] expressed a wavelength criterion to define whether the components vibrate in high frequency ranges. A structure is in the high frequency region if all components are long with respect to a wavelength of long members. The long members are defined as the component members that contain a large number of wavelengths and behave as flexible members. Currently, a widely used criterion for categorizing frequency ranges and for choosing between EFEA and other prediction techniques is the non-dimensional parameter $k\eta L$ [9,10], where k is the wavenumber, η is the damping loss factor and L is the characteristic length of the structure or acoustic space. When the shortest characteristic length $L_{\min}$ in the built-up structures meets $k\eta L_{\min} > 2$, the response prediction of the structure can be defined as a high frequency problem. The $k\eta L$ criterion is described under solid theoretical derivation by evaluating the validity of input power Π_{in} under a single driving force at the center of a simply supported beam [11]. In fact, for built-up structures such as beam-plate or beam-shell structures, as compared to plate or shell, the beam usually behaves as stiff member which should be regarded as in low-frequency range. The validity region and criterion of plates or shells that usually behave high frequency characteristics are more important and meaningful for designers in a certain sense. Wang [12] presented an accuracy analysis and applicability thresholds of EFEA and SEA for one and two-dimensional systems by quantitatively evaluating their boundary energy density error respectively. Based on the mathematical transformation of the modal criteria of SEA, Moens [13] presented a validity criterion in wave terms that the characteristic plate dimension must be 2.47 times higher than the shortest wavelength in the frequency of interest. This criterion is derived by studying the equality of potential energy and the kinetic energy, which is one of the approximations in the derivation of the EFEA governing equation.

One of the main assumptions of EFEA is that the elastic wave field in the structure is reverberant or quasi-reverberant, whereas the contribution of direct field is neglected. However, when the vibrational energy distribution over point excited plates with heavy damping is dealt with, the wave field is governed by direct field and does not obey the EFEA governing equation derived under reverberant plane wave assumption. The predictions by EFEA demonstrate significant estimate errors [14–16]. It is the main problem hindering the acceptance and spread of EFEA. Therefore, the study on the threshold of the formation of reverberant field is an appropriate aspect in validity region and criterion study of EFEA. The key to solving this issue is to describe the reverberant field and the direct field precisely and most importantly, to find a way that reveals the conversion relationship of the two fields. It could be solved by employing the radiative energy transfer method.

Radiative energy transfer method, proposed by Le Bot [17–20], is a recent wave method for the prediction of the structural response in high-frequency range. The main advantage of this method is the exact description of both the direct field and reverberant field. The equation of radiative energy transfer method also reveals the conversion relationship of these two fields, which is very useful for the validity study for EFEA. In addition, a validity criterion of SEA of diffuse field assumption is derived by Le Bot using that method [20,21]. Although there are significant differences between the reverberant field assumption of EFEA and the diffuse field assumption of SEA, as well as between the method SEA and EFEA themselves, there are similar frameworks of both fields that can be adapted. Therefore, the equation of radiative energy transfer method can be employed to study the validity region and criterion of EFEA from the formation of reverberant field.

The aim of this paper is to try to assess the validity region and criterion of EFEA theoretically by studying on the threshold for the formation of reverberant field. The paper is organized as follows. In Section 2, the basic concepts, assumptions and the theoretical basis of EFEA and radiative energy transfer method are introduced and discussed. In Section 3, the fundamental differences between the reverberant field assumption of EFEA and the diffuse field assumption of SEA are interpreted firstly. Based on the differences, the validity criterion of EFEA is studied by assessing the threshold for the formation of reverberant field. The equation of radiative energy transfer method is employed for the study. In Section 4, numerical simulations with different physical parameters are presented. Furthermore, the validity region and the diagrams of validity of EFEA are assessed. Discussions on the simulations results and the validity region are presented.

2. Basics and comparison of EFEA and radiative energy transfer method

The main assumptions, approximations and the theoretical basis that the EFEA and radiative energy transfer method share will be briefly overviewed in Section 2.1. The basic concepts, governing equations and the overview of the main limitations of EFEA and radiative energy transfer method are introduced respectively in Sections 2.2 and 2.3.

2.1. The common basics of EFEA and radiative energy transfer method

As the methods for the prediction of the vibrational response of built-up structures at high-frequency range, EFEA and radiative energy transfer method describe the dynamic response with energy variables. These energy variables are interpreted as the averaged results in a local sense to predict the space- and frequency-averaged vibrational behavior. The spatial average is performed over a wavelength, so as to cancel the spatial correlations within a wavelength, the spatial