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Force identification of dynamic systems using virtual work principle

Xun Xu*, Jinping Ou

School of Civil Engineering, Harbin Institute of Technology, Harbin 150090, China

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ABSTRACT

One of the key inverse problems for estimating dynamic forces acting on a structure is to determine the force expansion and the corresponding solving method. This paper presents a moving least square (MLS) method for fitting dynamic forces, which improves the existing traditional methods. The simulation results show that the force expansion order has a tiny effect on the types of forces, which indicates the MLS method's excellent ability for local approximation and noise immunity as well as good fitting function. Then, the differential equation of motion for the system is transformed into an integral equation by using the virtual work principle, which can eliminate the structural acceleration response without introducing the calculation error. Besides, the transformation derives an expression of velocity by integrating by parts, which diminishes the error propagation of the velocity. Hence, the integral equation of motion for the system has a strong constraint to noise with zero mean value. Finally, this paper puts forward an optimization method to solve the equation. The numerical stability can be enhanced as the matrix inversion calculation is avoided. Illustrative examples involving different types of forces demonstrate that the transformation of the differential equation proposed through virtual work principle can eliminate interference efficiently and is robust for dynamic calculation.

1. Introduction

Structural dynamic calculation and structural optimization are inseparable from the forces acting on the structure. The accuracy of the force characteristics influences structural security and construction cost directly and considerably. As is well known, engineering structures are complicated and endure great and variational acting forces (loads), such as wind force [1–4], seismic action [5,6], vehicle force [7–9], impact force [10,11], harmonic force [12], and so on. Consequently, a direct measurement of dynamic forces has been very difficult in the real working state of the structure. In the past decades, researchers were pursuing a feasible technique to accurately identify dynamic loads. Among the existing methods for identifying dynamic loads, the load reconstructed methods get more and more attention because they could be derived from a relatively easy measurement of structural dynamic response. The load reconstructed tasks include three main issues: the first is to obtain the expressions of the unknown acting forces involving many unknown parameters; the second is to solve the equations of motion for dynamic systems as an inverse problem; the third is to deal with random noise which has a

* Corresponding author.

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considerable effect on the stability of the solutions to the equations of motion for dynamic systems. The following text reviews the present research results for these tasks. In the statement below, force indicates acting force or load.

(1) Force identification demands obtaining of the force value at each moment. But, there are three difficulties to obtaining force values: (a) the number of loads needs to be identified; (b) the individual values in a load time history are generally unknown and (c) the sampling frequency is high. The difficulties lead to the great number of coupled equations to solve. At present, a common approach is that the force acting on the structure is approximately represented by a series of known primary functions. Thus, the force time history identification is transformed into identifying the combination coefficients of these primary functions, which greatly reduces the unknown number of equations in the force identification. Gunawan et al. [13] used B-spline function fitting impact load. The B-spline function has a high precision, but they did not clarify the node distance which is the key factor to improving the ill-posedness of the inverse problem. Yan et al. [14] expressed impact load as the sum of two sine waves, which transforms the identification of the impact force into an identification of wave amplitude and frequency. This method is quite effective in unimodal impact load, but this simplified method will produce a large error when the impact load has great amplitude and short-duration pulse. Obviously, the impact load mentioned above is simple and several primary functions fitting can achieve the ideal effect. Nevertheless, it is very different for arbitrary loads to further fitting and computational efficiencies by adopting different primary functions. The present research results have indicated that Chebyshev orthogonal basis functions as primary functions could bring a good fitting effect [15,16]. But, the disadvantages are that the force expansion order has to be pre-determined; besides, this method faces a problem of how to settle the conflict between computational efficiency and identification effect.
(2) As an inverse problem of structural mechanics, force identification is difficult to solve the differential equations of motion of the system directly [17]. Hence, a method was developed to simplify the dynamic differential equation, which could be divided into two kinds, frequency domain and time domain methods.
Through the Fourier transform, the differential equations can be transformed into algebraic equations and structural response can be expressed as the product of the frequency response function and external load. Furthermore, the external load can be obtained by the inverse frequency response function (IFSF) method [18,19]. The IFSF method has the advantages of simple calculation and distinct physical concept. However, a severe numerical instability exists in the IFSF method near the resonance frequency. Also, converting the time domain signals to the frequency domain using Fourier transform will introduce considerable error, when load duration is very short or load is a non-stationary signal.
The time domain method is widely applied at present, as it can be extended to the cases with load of short duration and different types of load application. Hu et al. [20] simplified the multi-degree-of-freedom (mdof) system using modal orthogonality and then calculated the responses of the individual sdof systems using the deconvolution method. But two common pitfalls are worthy of consideration sometimes, the modal truncation error and the instability of the solution from the ill-conditioned Toeplize matrix. Also using the modal superposition method, Shin [21] established the equation relationship between dynamic responses, generalized coordinates and the vibration model to identify the excitation force. However, Shin did not consider the effect of the structural damping; similarly, the modal truncation error affects the precision of the identification. Zhang et al. [22] transformed the load identification into a two-point boundary value problem; the problem with multiple inputs and outputs was then solved by Riccati matrix. However, noise was always induced when solving the Riccati matrix differential equation. Li et al. [23] established a virtual closed-loop feedback control system by connecting a virtual proportional feedback gain between the output and the structural model. The inverse problem is changed into the forward problem, namely, solving structural transient response. When the method was applied to the mdof system, the selection of modes and modal truncation had a great influence on the identification effect. Xu et al. [24] discussed the application of the independent component analysis method in load identification by using information entropy to measure the independence between the loads. But this method is only suitable for loads independent of each other. Yang et al. [25] put forward an objective function through minimizing the residual error between actual and computing responses, and then identified the actual force by a genetic algorithm. Nevertheless, the searching space of the genetic algorithm is so large that computing cost is typically high. Moreover, the crossover probability is just selected through trial and experience. The simulation results indicate that the genetic algorithm is accurate and reliable for harmonica forces, but it needs to further verify its accuracy and reliability for arbitrary force.
(3) The stability of the solution is one of the key problems to the inverse problem of force identification. Generally, ill-posed inverse problems require a numerical regularization to diminish the error of measured data, the error of discretization and round error. Commonly, Tikhonov regularization method [13] and truncated singular value decomposition (TSVD) method [26] are considered two good regularization methods in solving ill-posed problems. The Tikhonov regularization method employs well-posed problem solution to approximate the ill-posed problem of the solution. The key in Tikhonov regularization method is to select the regularization parameter α. Various methods were proposed to calculate the optimal regularization parameter, in which the L-curve method and the generalized cross validation (GCV) method are the most notable. Literature [27] reported that the L-curve method is more robust than the GCV method. For the TSVD method, the convergence of solution depends on the low index terms of the decomposition. The truncation at the index k means eliminating the 1st to the kth terms in the singular value decomposition. In fact, this index k is