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Proper orthogonal decomposition method for analysis of nonlinear panel flutter with thermal effects in supersonic flow

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ABSTRACT

The proper orthogonal decomposition (POD) method for analysis of nonlinear panel flutter subjected to supersonic flow is presented. Optimal POD modes are extracted from a chaotic Galerkin mode responses. The aeroelastic equations of motion are constructed using von Karman plate theory, first-order piston theory and quasi-steady thermal stress theory. A simply-supported plate with thermal loads from a uniformly distributed temperature is considered. Many types of panel behaviors, including stable flat, dynamically stable buckled, limit cycle oscillation, nonharmonic periodic motion, quasi-periodic motion and chaotic motion are observed. Our primary focus is on chaos and the route to chaos. It is found that a sudden transition from the buckled state to chaos occurs. Time history, phase portrait, Poincaré map, bifurcation diagram and Lyapunov exponent are employed to study chaos. The POD chaotic results obtained are compared with the traditional Galerkin solutions. It is shown that the POD method can obtain accurate chaotic solutions, using fewer modes and less computational effort than the Galerkin mode approach; additionally, the POD method converges faster in the analysis of chaotic transients. Effects of length-to-width ratios and thermal loads are presented. It is found that a smaller width for fixed length will produce more stable flutter response, while the thermal loads degrade the flutter boundary and result in a more complex evolution of dynamic motions. The numerical simulations show that the robustness of the POD modes depends on the dynamic pressure but not on temperature.

1. Introduction

Panel flutter is a self-excited oscillation created by the interactions of initial, elastic and aerodynamic forces [1]. In linear flutter theory, there is a critical velocity or dynamic pressure, beyond which the panel motion becomes unstable and grows exponentially with time. However, a nonlinear plate theory shows that when the panel oscillates with a deflection on the order of panel thickness, the panel not only bends but also stretches and thus nonlinear membrane stresses are induced and become significant. Such membrane forces restrain the panel motion to bounded-amplitude limit cycle oscillations (LCO). Hence, in the nonlinear panel flutter analysis, we seek to find these bounded-amplitude motions.

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Nomenclature

a, b	plate length, plate width, m
b_k	nondimensional modal amplitude (POD)
D	plate stiffness, N m
E	Young's modulus, N/m^2
h	plate thickness, m
k, l	mode number (POD)
K	number of modes retained (POD)
m, n	mode number (Galerkin)
M	number of modes retained (Galerkin)
Ma	Mach number
N	number of spatial points in a snapshot
N_x^T, N_y^T, N_{xy}^T	in-plane thermal force in x, y directions and in xy plane, N/m
$p-p_\infty$	aerodynamic pressure, N/m^2
q	$\rho U^2/2$, dynamic pressure, N/m^2
$\overline{\mathbf{Q}}$	snapshots matrix
r, s	mode number (Galerkin)
R_x^T, R_y^T	$N_x^T a^2/D$, $N_y^T a^2/D$, non-dimensional in-plane thermal loads
t	time, s
T	temperature differential, K
U	velocity, m/s
$\mathbf{V}$, $\mathbf{v_j}$	POD eigenvectors matrix, POD eigenvector
w	panel transverse deflection, m
x, y	streamwise, spanwise coordinate, m
α	thermal expansion coefficient, /C
β	$(Ma^2-1)^{1/2}$
λ_k^p	POD eigenvalue
ν	Poisson ratio
ρ, ρ_m	air density, plate density, kg/m^3
$\mathbf{\Phi}$, Φ	correlation matrix, Airy stress function
$\mathbf{\Psi}$, ψ_k	POD modes matrix, POD mode

Subscripts

p	peak

The aerodynamic heating cannot be ignored at high Mach number. An inclusion of thermal loads may generate in-plane thermal stresses for the panel, which will weaken the bending stiffness, leading to a degraded flutter boundary. An inclusion of thermal loads may result in various types of motions, including stable flat, dynamically stable buckled, LCO, nonharmonic periodic, quasi-periodic and chaotic motions.

Panel flutter studies started from 1950s, and many investigations have been done for analysis of nonlinear panel flutter [2–7]. Some reviews were done by Fung [8] and Dowell [9]. The methods for analysis used can be grouped into three categories: classical methods such as the Galerkin method and Rayleigh–Ritz method, finite element method, and reduced-order model. For a simply-supported plate [2], coupled nonlinear panel flutter modal equations were obtained using the partial differential equation (PDE)/Galerkin method, and then the direct time numerical integration was conducted to compute the LCO amplitudes. In addition, the PDE/Rayleigh–Ritz method using free–free beam and cantilever beam modes was applied to analyze the LCO of a cantilever plate [10,11]. For the same model, the chaotic motions were explored using the PDE/Rayleigh–Ritz method by Xie et al. [12]. However, the semi-analytical PDE/Galerkin or PDE/Rayleigh–Ritz approach has intrinsic limitations in dealing with complicated geometry or boundary conditions, or anisotropic material properties.

To conquer the drawbacks aforementioned, the finite element method (FEM) is proposed. It has been employed extensively to study the nonlinear panel flutter in the frequency domain [13] and the time domain [14–16]. Xue and Mei [13] applied a finite element frequency method to predict nonlinear flutter response of panels with arbitrary temperatures in supersonic flow, for both simply-supported and clamped conditions. The FEM frequency-domain method, however, can only study the LCO behavior. Cheng and Mei [16] presented a finite element time-domain formulation for hypersonic panel flutter analysis with thermal effects, considering simply-supported and clamped laminate panels. However, the major obstacles to implementation of the FEM time-domain formulation include (1) the large number of degrees of freedom (DOFs) that results in a large number of equations to solve; (2) the nonlinear stiffness matrices that have to be assembled and updated from the element nonlinear stiffness matrices at each time step; and (3) the time step of integration that has to be extremely small.

For DOFs reduction and computational savings, the reduced-order model (ROM) has been proposed and applied extensively by many researchers. Attar and Dowell [17] constructed a ROM using a system identification technique to investigate the LCO of a 45° delta wing. Guo and Mei [18,19] first used the aeroelastic modes for nonlinear panel flutter to reduce the number of ordinary differential equations (ODEs) and thus save computational costs. Amabili et al. [20,21] applied the POD method with a Galerkin projection (projection from the POD modes to the Galerkin modes) in a water-filled circular cylindrical shell. Epureanu et al. [22] employed a set of POD modes for the aeroelastic panels to construct a ROM and to identify the coherent structures of the panel dynamics [23]. More recently, Xie et al. constructed a POD reduced-order model (POD/ROM) for LCO analysis of a two-dimensional [5] and three-dimensional [6] simply-supported plates. For a cantilever plate [7], a projection-free POD method was employed. By application of the POD method, the number of governing equations of motion was reduced dramatically so that the computational costs were reduced significantly. However, the reduced-order model has not been available for the analysis of chaotic motions.

As noted by Moon [24], the motion of even very simple dynamical systems cannot always be predicted far into the future and these are labeled chaotic motions. We construct a reduced-order model based on the POD method for studies of chaos as well as other motions of a simply-supported panel with uniform temperature distribution due to aerodynamic heating. Time history, phase portrait, Poincaré map, bifurcation diagram and largest Lyapunov exponent (LLE) are employed.